Remarks/Arguments

Applicants thank the Examiner for considering this application. Applicants request reconsideration of this application in view the remarks to follow.

Claims 1-34 are pending in the application, with Claims 1, 11, 19, 25, 31, and 33 being the independent claims.

Based on the following remarks, Applicants respectfully request that the Examiner reconsider all outstanding rejections and that they be withdrawn.

At pages 3-6, the Office Action rejects Claims 1-5, 7, 10-15, 17, 19-21, 23, 25-27, and 29 under 35 U.S.C. § 102(e) as being anticipated by Lo et al. (U.S. Patent No. 6,987,958). Furthermore, at pages 7-10, the Office Action rejects various further claims under 35 U.S.C. § 103(a) as being unpatentable over Lo et al. in view of other references; in particular, Claims 6, 22, and 28 are rejected as being unpatentable over Lo et al. in view of Evans et al. (U.S. Patent Application Publication No. 2003/0083016), Claims 8, 9, and 16 are rejected as being unpatentable over Lo et al. in view of Nakamura (U.S. Patent No. 6,243,563), and Claims 18, 24, and 30 are rejected as being unpatentable over Lo et al. in view of Yoon (U.S. Patent No. 6,987,956). Applicants respectfully traverse these rejections for at least the following reasons.

Applicants initially note that the rejections of all of the claims mentioned above incorporate the rejections of independent Claim 1, 11, 19, or 25 under 35 U.S.C. § 102(e). Therefore, Applicants will address limitations of these independent claims, which will also serve to distinguish their respective dependent claims from the cited prior art.

Claims 1 and 11 include "a switch adapted to couple each of at least two receiver chains to one of at least two of plurality of antennas, ***said switch being further adapted to couple said receiver chains to said plurality of antennas such that <u>each receiver chain</u> is coupled to <u>a different one</u> of said plurality of antennas.***" (Emphasis added.) The Office Action asserts that Fig. 2, element 210 of Lo et al. corresponds to the claimed switch. However, it is respectfully submitted that this switch cannot correspond to the claimed switch because ***it does <u>not</u> couple each receiver chain to a <u>different</u> antenna.*** In particular, Applicants note that Fig. 2 includes element 205, which is described at col. 3, lines 7-16 as being an analog beamformer network. Element 205 is located between the antennas and element 210. As described at col. 3, lines 7-16, element 205 serves *to combine the various antenna outputs*, via a matrix, into a plurality of beams. It is these beams that are then switched by element 210. Therefore, element 210 does not couple each of at least two receiver chains to a <u>different one</u> of a plurality of antennas, as recited in these claims.

To elaborate further, Lo et al. at col. 3, lines 7-40 reads as follows:

> The N-element antenna array 201 is coupled to N-by-N analog beamformer 205. The beamformer is a multiple-beamformer network such as the one known in the art as a Butler matrix described in "Digital, Matrix, and Intermediate Frequency Scanning" by L.J. Butler, in R.C. Hansen, ed. *Microwave Scanning Arrays*, Academic Press, New York, 1966. That matrix uses hybrid junctions and fixed phase shifters to create N beams from the N antenna outputs. Thus, the output of beamformer 205 is shown as beams b_1 to b_N. All of these beams, which can be orthogonal beams, are inputs to an exclusion logic N-to-M switch network 210. The switch network receives all N beams and, based on switching control signals from a digital signal processor 230, selects M of those beams for processing by a plurality of receivers. One beam is selected for transfer to the primary transceiver 215 and the remaining M-1 selected beams are provided to the auxiliary receivers shown together as element 220 in FIG. 2. The receivers then produce output signals which constitute received signals from

> the various produced beams, x_1 to x_M. These output signals from the receivers are provided to the digital signal processor (DSP) 230 which assigns weights to the received signals and then combines them in accordance with the digital signal processing algorithm, stored within the processor or in an adjunct memory, to provide an output signal y. That output signal is subsequently demodulated by the modulator/demodulator 240 to create a binary stream which includes the message received from the transmitter. By manipulation of the switching network configuration under control of the DSP and by the selection of multiple beams for processing, the present invention can improve the signal-to-noise ratio of the system by emphasizing the impact of beams that are constructive to the process and de-emphasizing the impact of beams that are not constructive to the process.

Lo et al., col. 3, lines 7-40. It is, thus, noted that switch 210 is situated following beamformer 205, which is directly connected to the antennas 201 in Figure 2 of Lo et al. As discussed in the above passage, element 205 is a ***beamformer***. It is well known in the art that a beamformer takes the inputs of multiple antennas and ***combines them into one or more output beams***. Therefore, ***it <u>cannot</u> be stated that a particular beam (beamformer output) corresponds to the output of any one antenna providing input to the beamformer.*** This is supported by the various attached exhibits, which describe various beamformers, including the Butler matrix specifically discussed in the cited passage. In view of these references, it is apparent that the essence of a beamformer is ***to combine antenna outputs***. If the antenna outputs are combined to form the beams, then a switch that connects the beams to receivers is connecting components of outputs of multiple antennas to each receiver.

In view of this, switch network 210 ***cannot*** couple the receiving elements 215, 220 to the plurality of antennas 201 ***such that each receiving element 215, 220 is coupled to a <u>different</u>***

one of the antennas 201. In view of this, the system of Lo et al. cannot be said to read on the invention as claimed in Claims 1 and 11.

For at least these reasons, Claims 1 and 11, as well as their dependent claims, Claims 2-10 and 12-18, respectively, are allowable over the cited prior art.

Claims 19 and 25 similarly recite that each of a number of receiver chains receives a different signal from a different selected antenna. Therefore, similarly, Lo et al. does not anticipate these claims, for the same reason (i.e., Lo et al. does not disclose all elements of these claims).

For at least these reasons, Claims 19 and 25, as well as their dependent claims, Claims 20-24 and 26-30, respectively, are allowable over the cited prior art.

At page 6, the Office Action rejects Claims 31 and 33 under 35 U.S.C. § 102(e) as being anticipated by Yoon (U.S. Patent No. 6,987,956). At pages 10-11, the Office Action rejects Claims 32 and 34 under 35 U.S.C. § 103(a) as being unpatentable over Yoon in view of Tsien et al. (U.S. Patent Application Publication No. 2003/0166394). Applicants respectfully traverse these rejections for at least the following reasons.

Applicants note that the rejections of all of Claims 32 and 34 incorporate the rejections of independent Claims 31 and 33, respectively, under 35 U.S.C. § 102(e). Therefore, Applicants will address limitations of these independent claims, which will also serve to distinguish their respective dependent claims from the cited prior art.

Claims 31 and 33 recite "adjusting a data rate of a signal transmitted by a first transceiver employing diversity combining ***to compensate for a lack of use of diversity combining <u>at a</u>***

<u>second transceiver.</u>" (Emphasis added.) Yoon, noting col. 3, lines 23-51 and Table 2, discusses a transceiver system that is switchable between high data rate and low data rate modes of operation. It is further discussed that the high data rate mode uses a helical antenna and a whip antenna, while the low data rate mode uses only the helical antenna. However, it is noted that the use of a different data rate disclosed at the transceiver of Yoon corresponds to the number of antennas used ***at that same transceiver. Nowhere in Yoon is it disclosed that switching of the data rate of the transceiver is performed to compensate for the lack of use of diversity combining <u>at a second transceiver</u>***, as claimed. For at least this reason, Yoon does not anticipate Claims 31 and 33.

For at least these reasons, it is respectfully submitted that Claims 31 and 33, as well as their respective dependent claims, Claims 32 and 34, are allowable over the cited prior art.

Applicants respectfully state that the above discussion is not to be understood as including all possible arguments, and therefore, Applicants' electing not to address a particular element of the Office Action should not be understood as indicating concurrence with the characterizations of the claims or of the cited prior art found in the Office Action. Applicants acknowledge that there may be additional reasons for which the various claims are allowable over the cited prior art, and Applicants' election not to address such reasons at this time should not be understood as a concurrence with the Office Action or as a waiver of the right to argue such reasons at a later date.

Conclusion

All of the stated grounds of rejection have been properly traversed, accommodated, or rendered moot. Applicants, therefore, respectfully request that the Examiner reconsider all presently outstanding rejections and objections and that they be withdrawn. Applicants believe that a full and complete reply has been made to the outstanding Office Action and, as such, the present application is in condition for allowance. If the Examiner believes, for any reason, that personal communication will expedite prosecution of this application, the Examiner is hereby invited to telephone the undersigned at the number provided.

Prompt and favorable consideration of this Response is respectfully requested.

Respectfully submitted,

Date: July 13, 2006

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Beamforming

From Wikipedia, the free encyclopedia

Beamforming is a signal processing technique used with arrays of transmitters or receivers that controls the directionality of, or sensitivity to, a radiation pattern. When receiving a signal, beamforming can increase the gain in the direction of wanted signals and decrease the gain in the direction of interference and noise. When transmitting a signal, beamforming can increase the gain in the direction the signal is to be sent. This is done by creating beams and nulls in the radiation pattern. Beamforming can also be thought of as spatial filtering.

Beamforming takes advantage of interference to change the directionality of the array. When transmitting, a beamformer controls the amplitude and phase of the signal at each transmitter, in order to create a pattern of constructive and destructive interference in the wavefront. When receiving, information from different sensors is combined in such a way that the expected pattern of radiation is preferentially observed.

For example, to send a sharp pulse of sound towards a ship in the distance, simply transmitting that sharp pulse from every speaker in an array simultaneously fails because the ship will first hear the pulse from the speaker that happens to be nearest the ship, then later pulses from speakers that happen to be the further from the ship. The beamforming technique involves sending the pulse from each speaker at slightly different times (the speaker closest to the ship last), so that every pulse hits the ship at exactly the same time, producing the effect of a single strong pulse from a single powerful speaker.

To detect the clank of someone dropping a wrench, the beamforming technique involves combining delayed signals from each antenna at slightly different times (the antenna closest to the wrench has the longest delay), so that every clank reaches the headphones at exactly the same time, making one loud clank, as if the signal came from a single, very sensitive antenna.

The time delay is sometimes called a "phase shift", so the array of antennas, each one delayed a slightly different amount, is called a phased array.

The signal from each antenna is amplified by a different "weight." When that weight is negative by just the right amount, the faint clank received by that antenna can exactly cancel out the faint clank received by some other antenna, causing a "null." This is useful to ignore noise or jammers in one particular direction, while listening for events in other directions.

For the full mathematics on directing beams using amplitude and phase shifts, see the math section in phased array.

Beamforming techniques can be broadly divided into two categories:

- conventional (fixed) beamformers
- adaptive beamformers

Conventional beamformers use a fixed set of weightings and time-delays to combine the signals from the sensors in the array, primarily using only information about the location of the sensors in space and the wave directions of interest. In contrast, adaptive beamforming techniques, generally combine this information with properties of the signals actually received by the array, typically to improve rejection of unwanted signals from other directions. As the name indicates, an adaptive beamformer is able to automatically adapt its response to different situations.

This term has been frequently used in sensor networks.

Sonar beamforming

Sonar implementation is comparatively easy compared to electromagnetic implementation because of the relatively slow speed of sound in relation to the speed of the controlling hardware. Sonar arrays are used both actively and passively in 1, 2, and 3 dimensional arrays.

- 1 dimensional arrays are usually towed behind ships.
- 2 dimensional arrays are common in side-scan sonar.
- 3 dimensional arrays are used in 'sonar domes' in the modern submarine.

Sonar also differs from radar in that all directions can be listened to and, in theory, broadcast to, simultaneously, and the phases can be manipulated entirely by signal processing software, as compared to present radar systems that use hardware to 'listen' in a single direction at a time. At a certain level the human brain does the same thing, using signal processing on its 2 dimensional array (ears) to figure out where sound comes from.

See also

- Phased array antennas, which uses beamforming to steer the beam
- aperture synthesis
- synthetic aperture radar
- synthetic aperture sonar
- inverse synthetic aperture radar (ISAR)
- side-scan sonar

External links

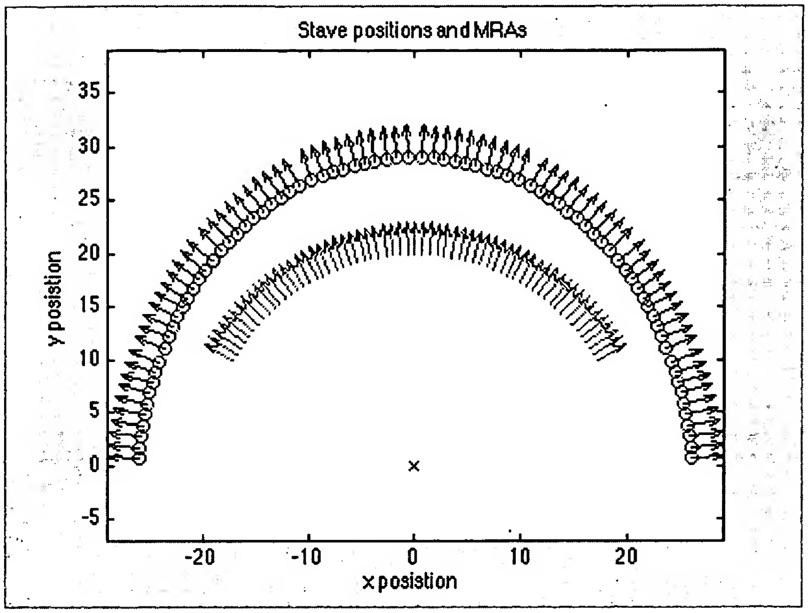
- "How to create beam-forming smart antennas using FPGAS" (http://www.embedded.com/showArticle.jhtml?articleID=60401726)by Deepak Boppana and Asif Batada in *Embedded Systems Programming* 2005-02-17
- "A Primer on Digital Beamforming" (http://www.spectrumsignal.com/publications/beamform_primer.pdf) by Toby Haynes, March 26, 1998
- [1] (http://www.ece.utexas.edu/~allen/Beamforming)

Retrieved from "http://en.wikipedia.org/wiki/Beamforming"

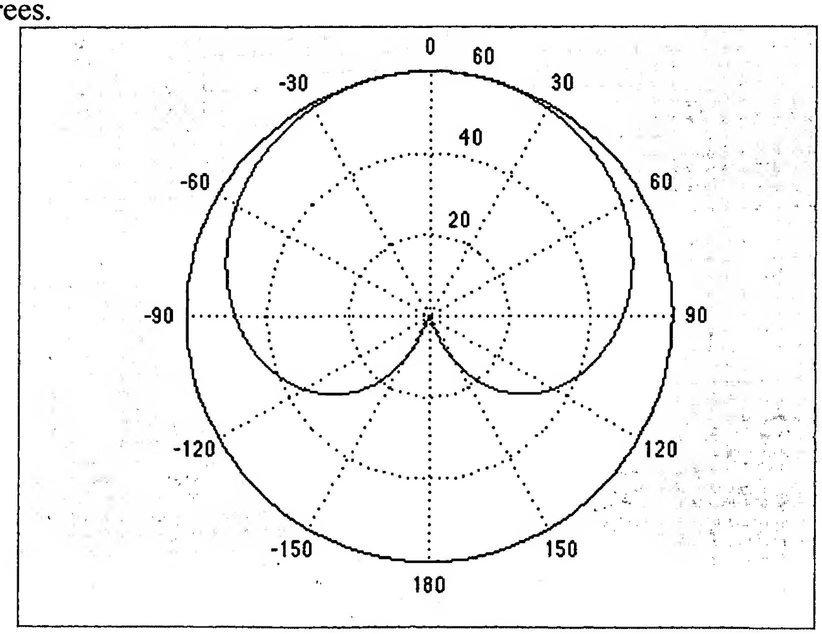
Categories: Electronics | Radar

What Is Beamforming?

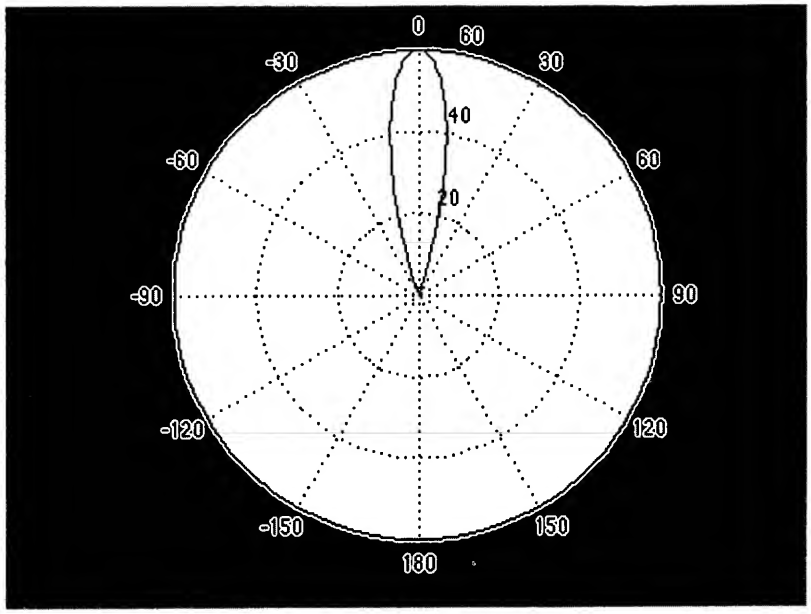
A beamformer is a spatial filter that operates on the output of an array of sensors in order to enhance the amplitude of a coherent wavefront relative to background noise and directional interference. The figure below shows a curved array of hydrophone sensors, or staves. Each sensor (red circle) is located at an (x,y) coordinate as shown. These sensors are pointed in known directions (blue arrows), and we wish to form beams which point in chosen directions (green arrows). The "pointing direction" is called the Maximum Response Angle (MRA), and can be arbitrarily chosen for the beams.



The response of a given element is plotted on a polar graph, where the angle is the offset from the MRA, and the radius is the magnitude response (dB) in that direction. Element responses (determined by the 3dB down point) are very wide -- in this example the width is about 90 degrees.

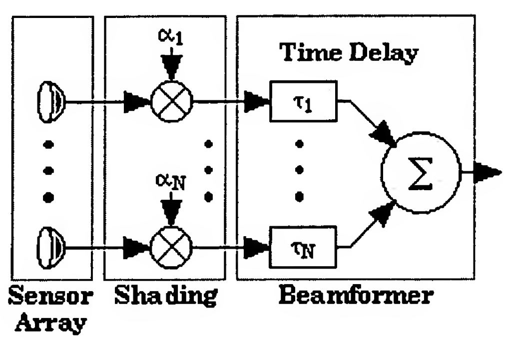


The goal of beamforming is to sum multiple elements to achieve a narrower response in a desired direction (the MRA). That way when we hear a sound in a given beam, we know which direction it came from. Real implementations introduce things such as nulls and sidelobes, which we won't discuss here.

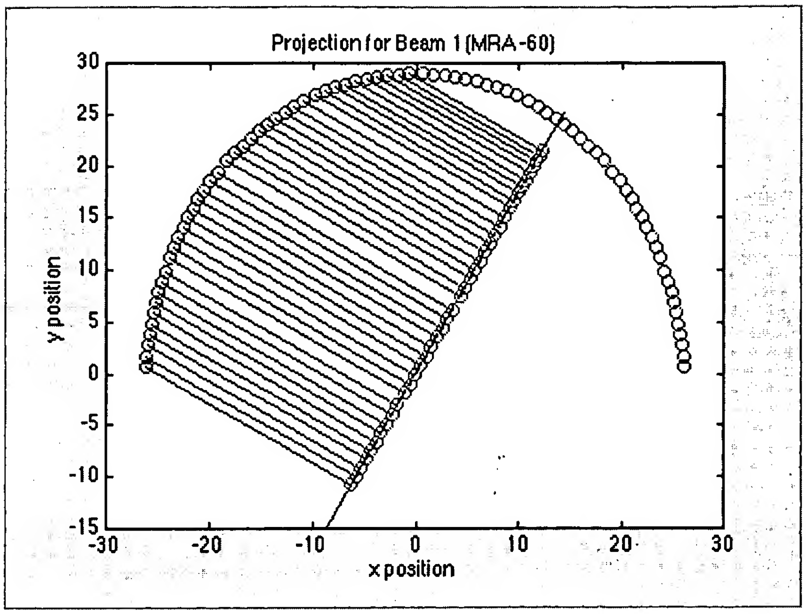


Implementing a Beamformer

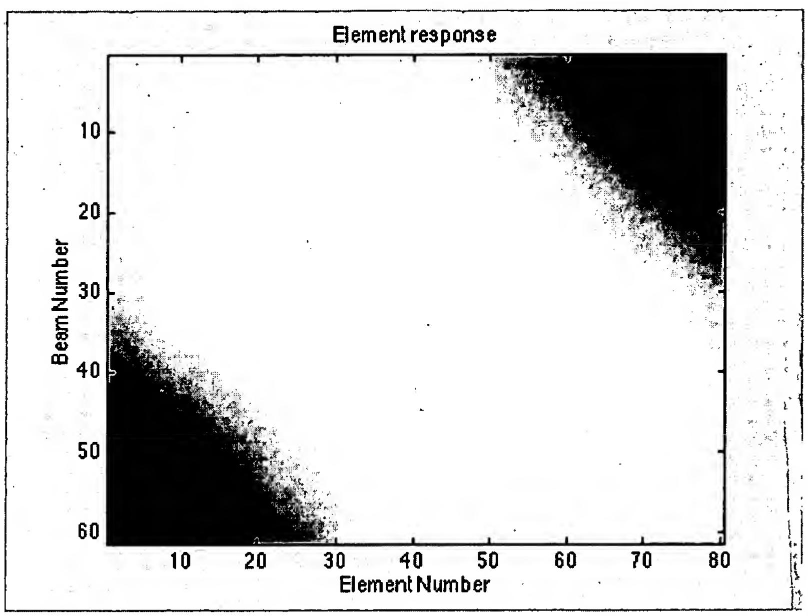
Time-domain beamforming is done by delaying and adding shaded outputs from an array of transducers. The (optional) shading of the sensor outputs is done to improve the spatial response characteristics of the beam, and is roughly equivalent to "windowing" in DSP theory. Each beam is formed by delaying and summing sensor elements. The following block diagram shows how a single beam is formed from N transducers, in an analog beamformer.



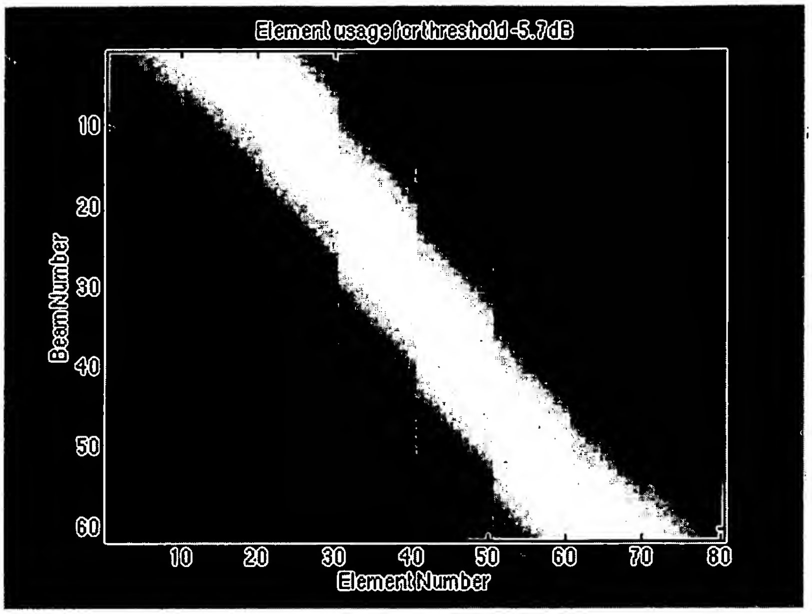
The delay used for each sensor element is determined by array geometry and the desired MRA. Projecting the elements onto a line which is perpendicular to the beam's MRA gives a distance for each element. This distance (divided by the speed of sound) gives the delay required to form the beam at the desired MRA.



Note that we did not use every sensor to calculate every beam. Since our array is curved, each sensor contributes to each beam differently. In the following plot, high responses are white and low responses are black. If the element and the beam point in the same direction, the response is high.

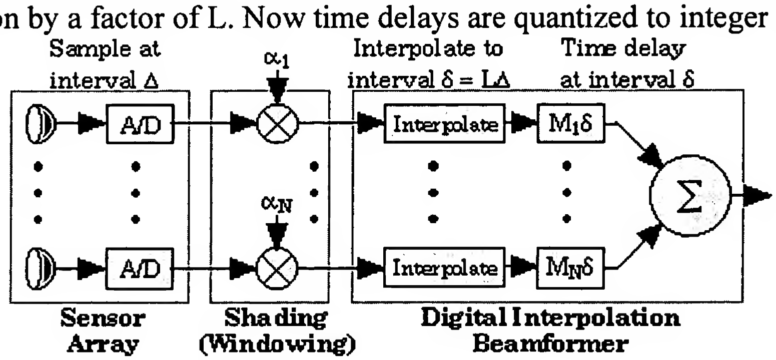


It doesn't make sense to use elements which point in the wrong direction. In the following plot, any element which is some fixed threshold below its maximum is zeroed out. Now we're only using the non-zero elements to form each beam. This step saves on processing, with minimal beam degradation.

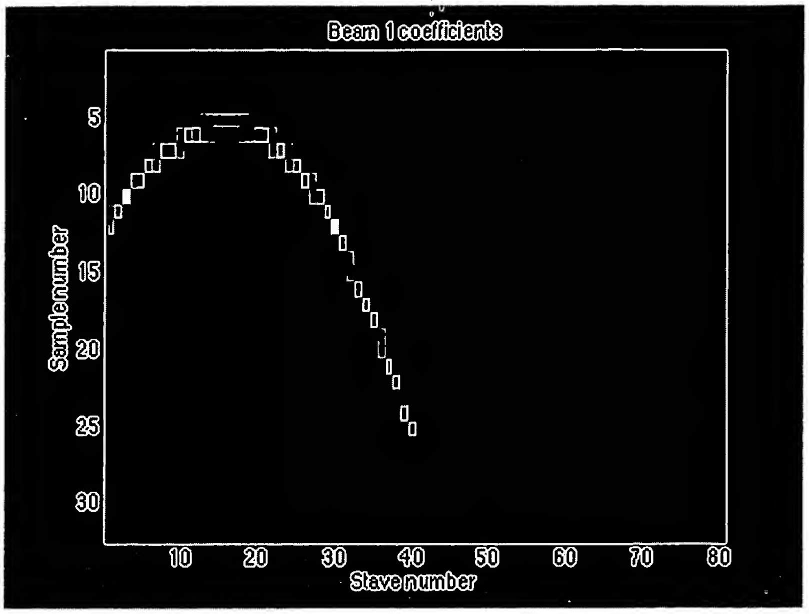


Digital Interpolation Beamforming

In a digital implementation, we sample these elements at a rate just above the Nyquist criterion. Although this preserves the frequency content of the signal, this does not give enough steering delay resolution. Digital interpolation is performed, increasing the steering-delay resolution by a factor of L. Now time delays are quantized to integer sample delays.



In this example we use unity shading, and simply interpolate across two samples. As a result, all coefficient values are between zero and one. The figure below plots these values, where white is one and black is zero. Non-zero coefficients are extremely sparse, allowing efficient implementation. Note that each "picture" contains the values required to calculate one sample of one beam output.



Beamforming as a Sparse FIR Filter

Modeling the beamformer as a FIR filter allows for a simple, concise organization of the algorithm. For our model we use the following parameters, with values from the example in parentheses:

- T - the total number of elements in the array (80)
- D - the maximum sample delay due to array geometry (31)
- L - the length of the interpolation filter (2)
- B - the number of beams calculated (61)
- S - the number of staves (elements) used to calculate a beam (50)

If multiple samples of the entire sensor array are stored contiguously in memory, then each beam's coefficients can be represented by a FIR filter of length N = (D+L-1)T. Now the entire beamforming operation (for one sample of B beams) can be represented by a single operation:

$$\underset{(1\text{ by }N)}{\begin{bmatrix} \text{Incoming Data} \end{bmatrix}} \times \underset{(N\text{ by }B)}{\begin{bmatrix} \text{Beam 1 coefs} & \cdots & \text{Beam B coefs} \end{bmatrix}} = \underset{(1\text{ by }B)}{\begin{bmatrix} \text{Beam Data (1 sample)} \end{bmatrix}}$$

The FIR filter length, N, can be extremely long -- in our example it is 2560. However, the number of non-zero coefficients is only 100, for a sparsity of 96%. As a result, 6100 multiply-accumulates (MACs) are required per sample. At high-frequency sonar sample rates, we are approaching one billion MACs per second.

Coefficient filter length $N = (D + L - 1) T$

Non-zero coefficients $C = L S$

$$\text{Sparsity} = 1 - \frac{C}{N}$$

$$\text{MACs per sample} = B\,C$$

For additional information:

- R. G. Pridham and R. A. Mucci, "A Novel Approach to Digital Beamforming." *Journal of the Acoustical Society of America*, vol. 63, no. 2, pp. 425-434, Feb. 1978.
- R. G. Pridham and R. A. Mucci, "Digital Interpolation Beamforming for Low-Pass and Bandpass Signals." *Proceedings of the IEEE*, vol. 67, no. 6, pp. 904-919, June 1979.
- R. A. Mucci, "A Comparison of Efficient Beamforming Algorithms." *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. ASSP-32, no. 3, 548-558, June 1984.
- J. Stine, "Beamforming Illustrations" report and slides, for EE381K Advanced Digital Signal Processing, July 1997.
- Brian Evans' digital filtering web page.

Page by Greg Allen <gallen@arlut.utexas.edu>

A Primer on Digital Beamforming

Toby Haynes, Spectrum Signal Processing
March 26, 1998

Introduction

Beamforming is the combination of radio signals from a set of small non-directional antennas to simulate a large directional antenna. The simulated antenna can be pointed electronically, although the antenna does not physically move. In communications, beamforming is used to point an antenna at the signal source to reduce interference and improve communication quality. In direction finding applications, beamforming can be used to steer an antenna to determine the direction of the signal source.

This introduction to beamforming covers the basic properties of antennas and antenna arrays, then explains how beamformers are built using digital radio hardware and DSP's. Super-resolution direction finding is also explained.

Antennas and Wavelength

An antenna for a radio transmitter converts electrical signals on a cable, from the transmitter, into electromagnetic waves. The antenna consists of electrical conductors (wires, pipes, reflecting surfaces, etc) that create electric and magnetic fields in the space around them. If the fields are changing, they propagate outward through space as an electromagnetic wave at the speed of light.

Speed of Light $\mathbf{c = 3x10^8}$ **meters/sec**

Any antenna that transmits can also receive. Passing electromagnetic waves excite currents in the antenna's conductors. The antenna captures some energy from passing waves and converts it to an electrical signal on the cable.

When designing an antenna, its dimensions are specified in terms of the ***wavelength*** of the radio signal being transmitted or received. Wavelength is the distance from the beginning of one electromagnetic wave cycle to the next.

$$\lambda = c / f_c$$

λ is wavelength in meters
f_c is the carrier frequency of the radio signal in Hz
c is the speed of light ($3x10^8$ meters/sec)

Wavelengths For Common Radio Signals

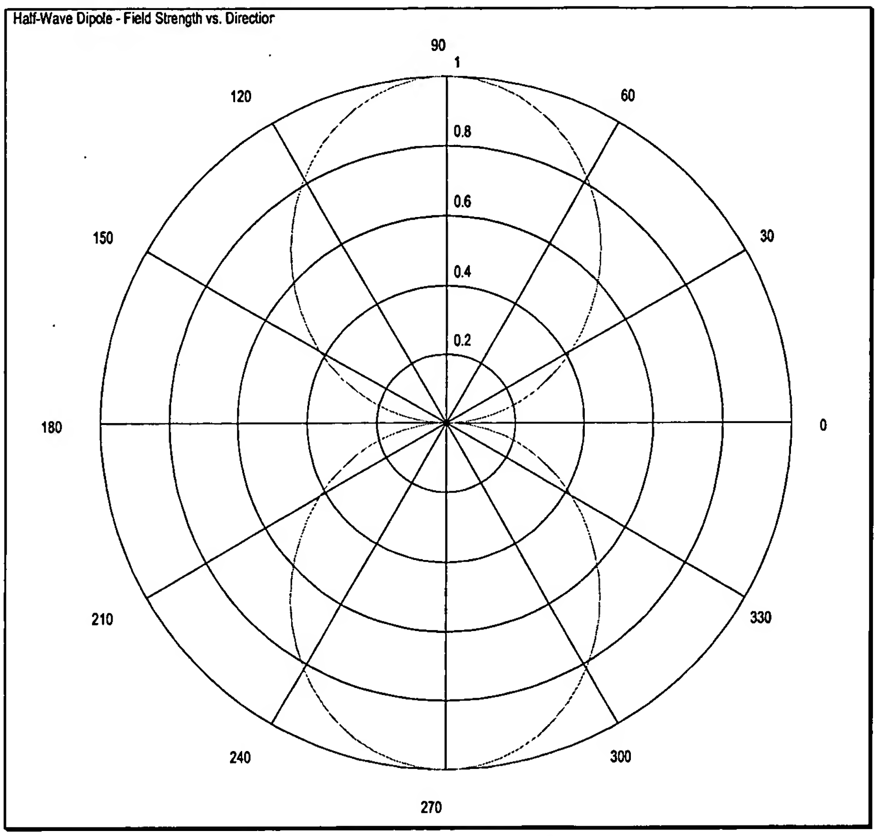
Signal	Frequency	Wavelength
AM Radio	1 MHz	300 meters
FM Radio	100 MHz	3 meters
Cellular Telephone	850 MHz	35 cm
Cellular PCS	1,800 MHz	17 cm
X-Band Radar	10,000 MHz	3 cm

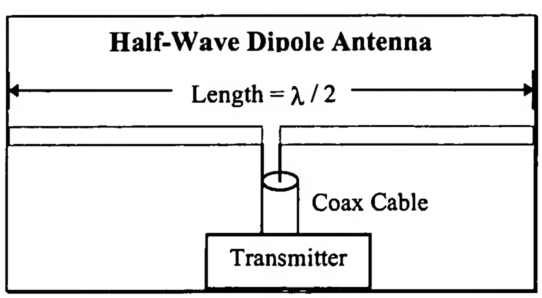
Antenna Radiation Patterns

A transmitting antenna generates stronger electromagnetic waves in some directions than others. A plot of field strength vs. direction is called the antenna's ***"radiation pattern."*** It's always the same for receiving as for transmitting.

An electromagnetic wave measured at a point far from the antenna is the sum of the radiation from all parts of the antenna. Each small part of the antenna is radiating waves of a different amplitude and phase, and each of these waves travels a different distance to the point where a receiver is located. In some directions, these waves add constructively to give a gain. In some directions they add destructively to give a loss.

A half-wave dipole is a simple antenna that consists of a half wavelength of wire, cut in the center for connection of the cable. The following figure shows its radiation pattern.

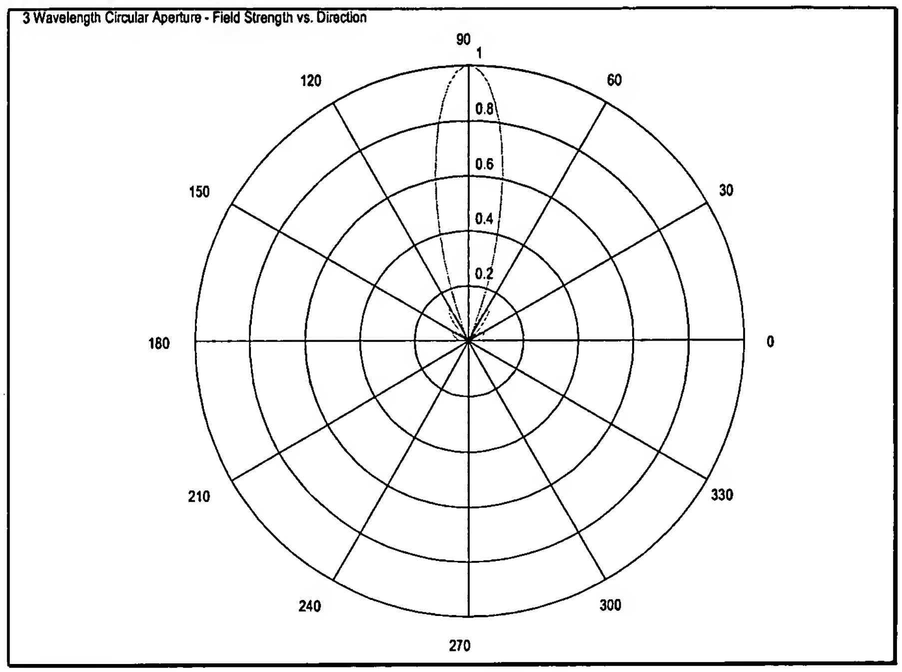


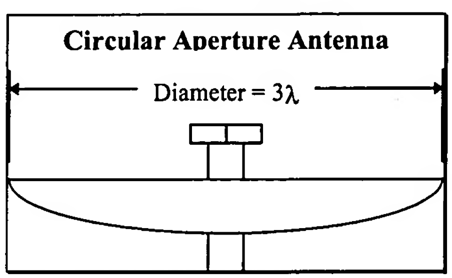


Directional Antennas

A directional antenna is one designed to have a gain in one direction and a loss in others. An antenna is made directional by increasing its size. This spreads the radiating conductors of the antenna over a larger distance, so that the constructive and destructive interference can be better controlled to give a directional radiation pattern.

A satellite dish antenna can, simplistically, be considered a circular surface that radiates electromagnetic waves equally from all parts. It has a narrow central ***"beam"*** of high gain, as shown in the following figure, that is aimed at the satellite. As the dish diameter, in wavelengths, is increased the central beam gets narrower. Notice the smaller beams, called ***"side lobes"***, on either side of the central beam. Directions in which the signal strength is zero are called "***nulls.***"

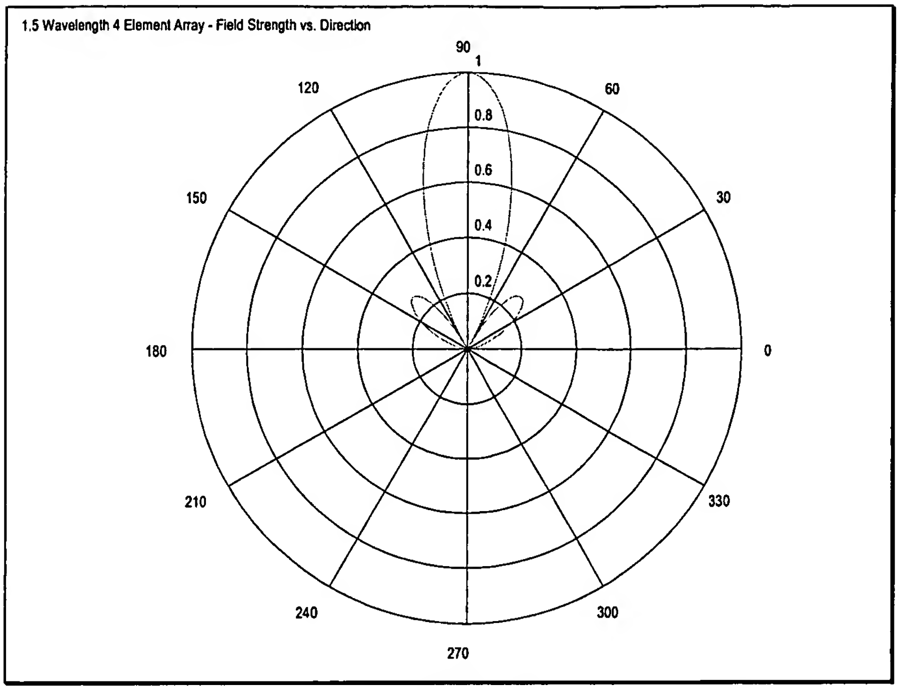


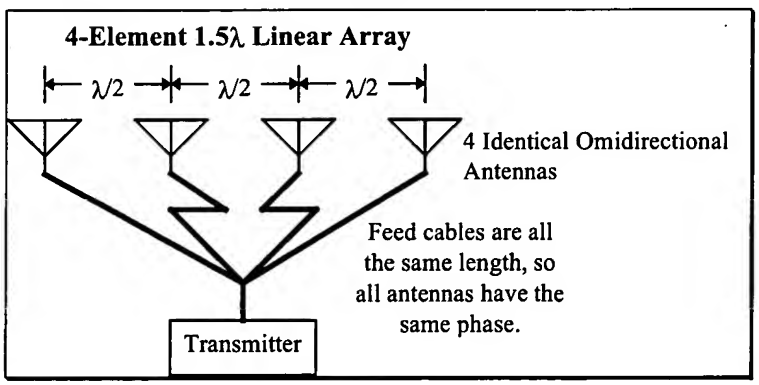


Linear Arrays

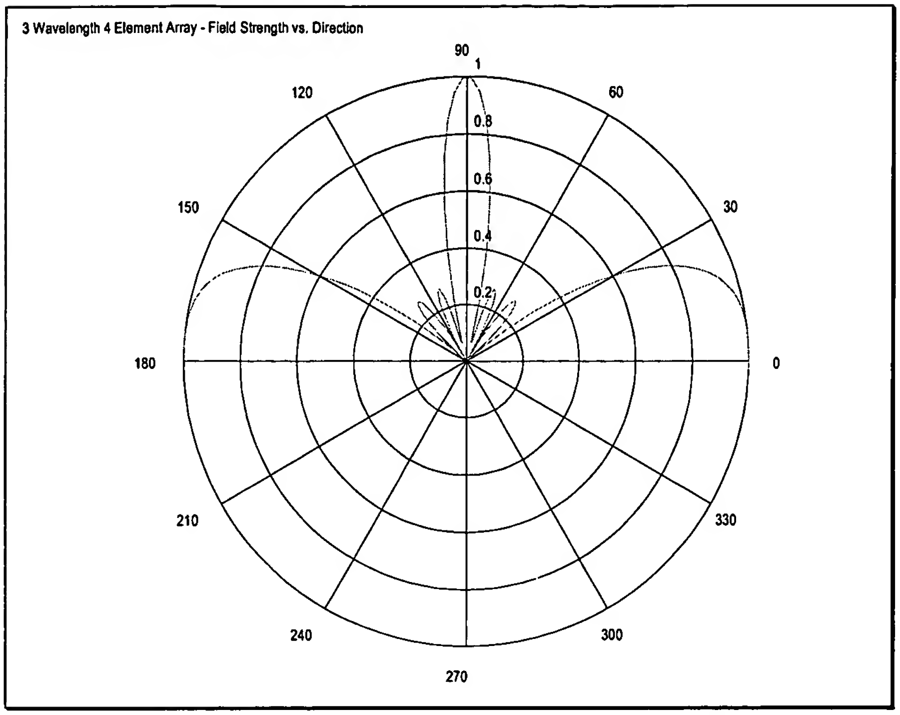
A simple directional antenna consists of a linear array of small radiating antenna elements, each fed with identical signals (the same amplitude and phase) from one transmitter. As the total width of the array increases, the central beam becomes narrower. As the number of elements increases, the side lobes become smaller.

The following figure is the radiation pattern for a line of 4 elements (small antennas) spaced 1/2 wavelength apart.

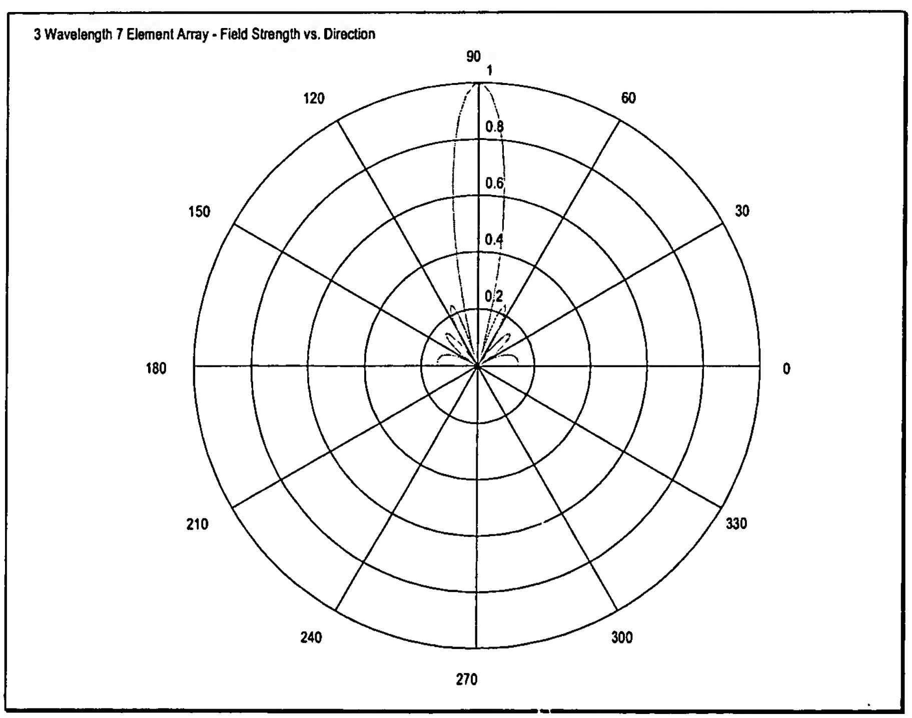


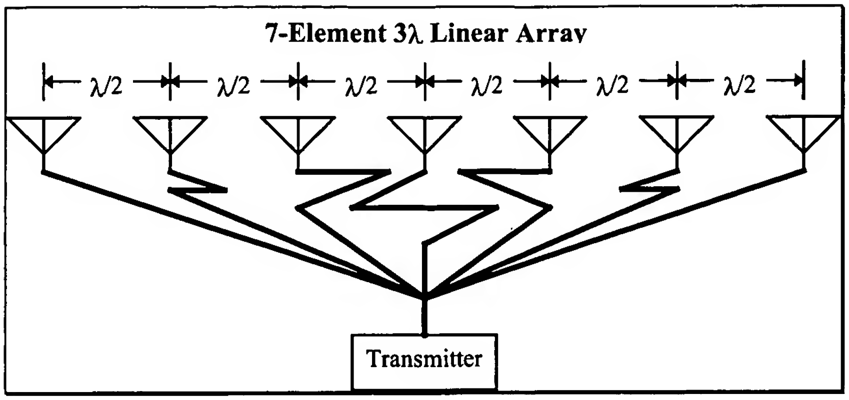


If the spacing is increased to more than 1/2 wavelength, large side lobes begin to appear in the radiation pattern. However, the central beam gets narrower because the overall length of the antenna has increased. The following radiation pattern, for 4 elements spaced 1 wavelength apart, illustrates this.



By keeping the overall length the same, and adding elements to reduce the spacing back to 1/2 wavelength, the side lobes are reduced. Following is the radiation pattern if 3 more elements are added to the antenna above to reduce the element spacing.

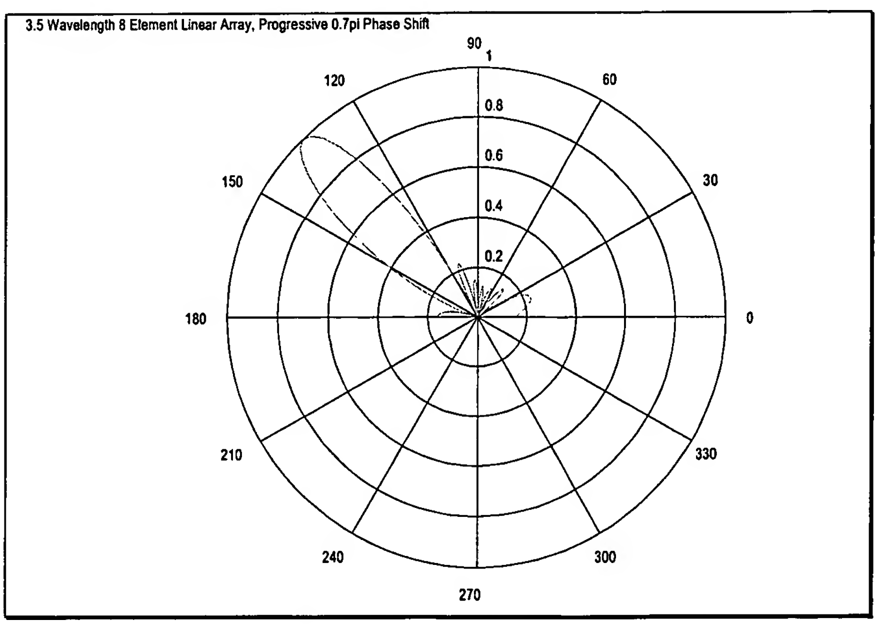


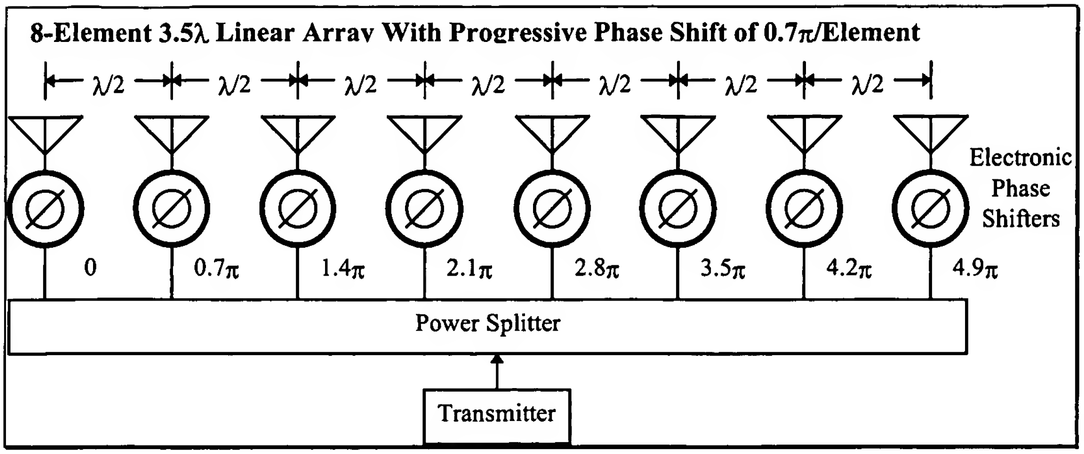


Electronically Steered Arrays

By varying the signal phases of the elements in a linear array, its main beam can be steered. The simplest way of controlling signal phase is to systematically vary the cable lengths to the elements. Cables delay the signal and so shift the phase. However, this does not allow the antenna to be dynamically steered.

In an electronically steered array, programmable electronic phase shifters are used at each element in the array. The antenna is steered by programming the required phase shift value for each element. The beam pattern below is for an 8-element linear array with a progressive phase shift of 0.7π per element. The central beam has been steered about 45 degrees to the left. A phase shift of 2π corresponds to one wavelength or one carrier wave period, and more positive values are equivalent to saying that the signal is transmitted earlier.





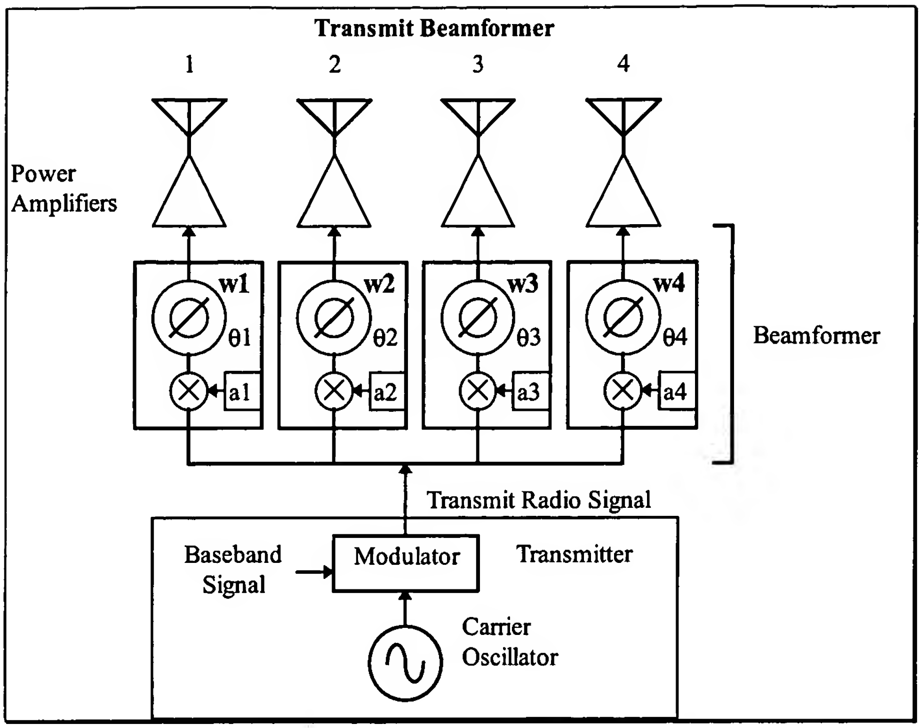
Array Configurations

An antenna array does not need to be linear. Often, antenna elements are arranged in a circle so that the array can form beams equally well in all directions. On vehicles, antenna elements may be placed in any convenient locations and at different heights to form a 3-dimensional array. For these arrays, determining phase shifts to steer the antenna is more complicated than for linear arrays.

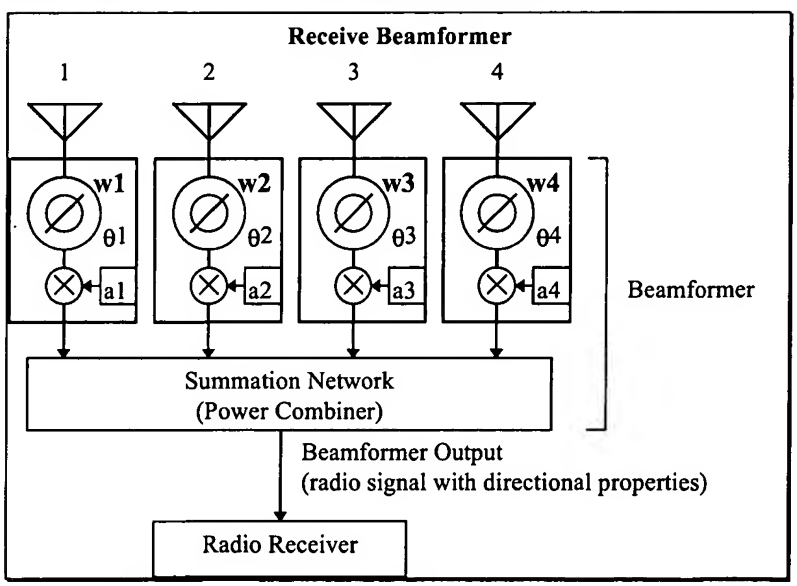
Beamforming

In beamforming, both the amplitude and phase of each antenna element are controlled. Combined amplitude and phase control can be used to adjust side lobe levels and steer nulls better than can be achieved by phase control alone. The combined relative amplitude $\mathbf{a}_k$ and phase shift θ_k for each antenna is called a "***complex weight***" and is represented by a complex constant w_k (for the k^{th} antenna).

A beamformer for a radio transmitter applies the complex weight to the transmit signal (shifts the phase and sets the amplitude) for each element of the antenna array.



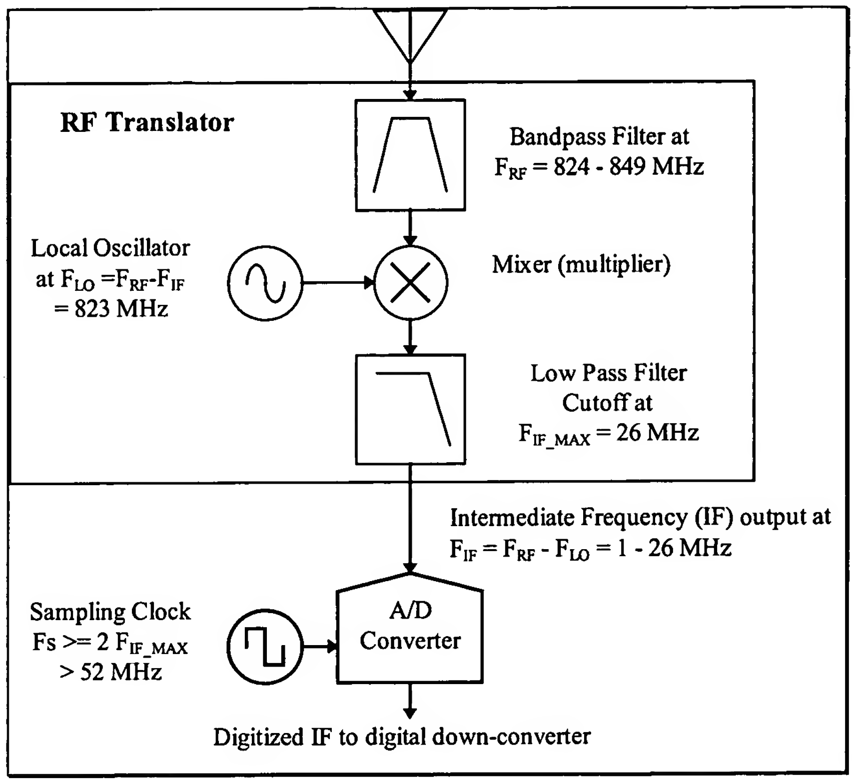
A beamformer for radio reception applies the complex weight to the signal from each antenna element, then sums all of the signals into one that has the desired directional pattern.



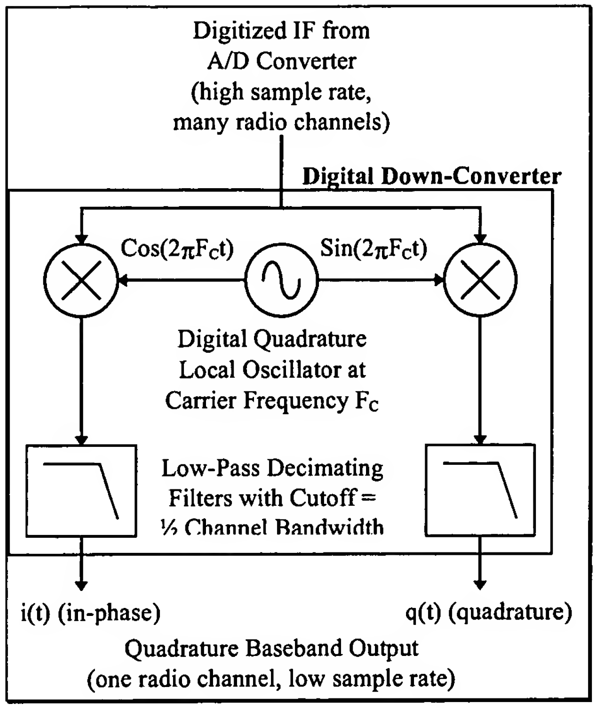
Digital Beamforming

In digital beamforming, the operations of phase shifting and amplitude scaling for each antenna element, and summation for receiving, are done digitally. Either general-purpose DSP's or dedicated beamforming chips are used.

The rest of this discussion focuses on beamforming receivers. Digital processing requires that the signal from each antenna element is digitized using an A/D converter. Since radio signals above shortwave frequencies (>30 MHz) are too high to be directly digitized at a reasonable cost, digital beamforming receivers use analog "**RF translators**" to shift the signal frequency down before the A/D converters. The following figure shows a translator that shifts the entire cellular telephone uplink band at 824-849 MHz down to the 1-26 MHz range.



Once the antenna signals have been digitized, they are passed to "digital down-converters" that shift the radio channel's center frequency down to 0 Hz and pass only the bandwidth required for one channel. The down-converters produce a "**quadrature**" baseband output at a low sample rate.



The quadrature baseband i and q components can be used to represent a radio signal as a complex vector (phasor) with real and imaginary parts. Two components are required so that both positive and negative frequencies (relative to the channel center frequency) can be represented.

$$\mathbf{s(t) = x(t) + j\ y(t)}$$

s(t) is the complex baseband signal
x(t) = i(t) is the real part
y(t) = -q(t) is the imaginary part
j is $\sqrt{(-1)}$

For beamforming, the complex baseband signals are multiplied by the complex weights to apply the phase shift and amplitude scaling required for each antenna element.

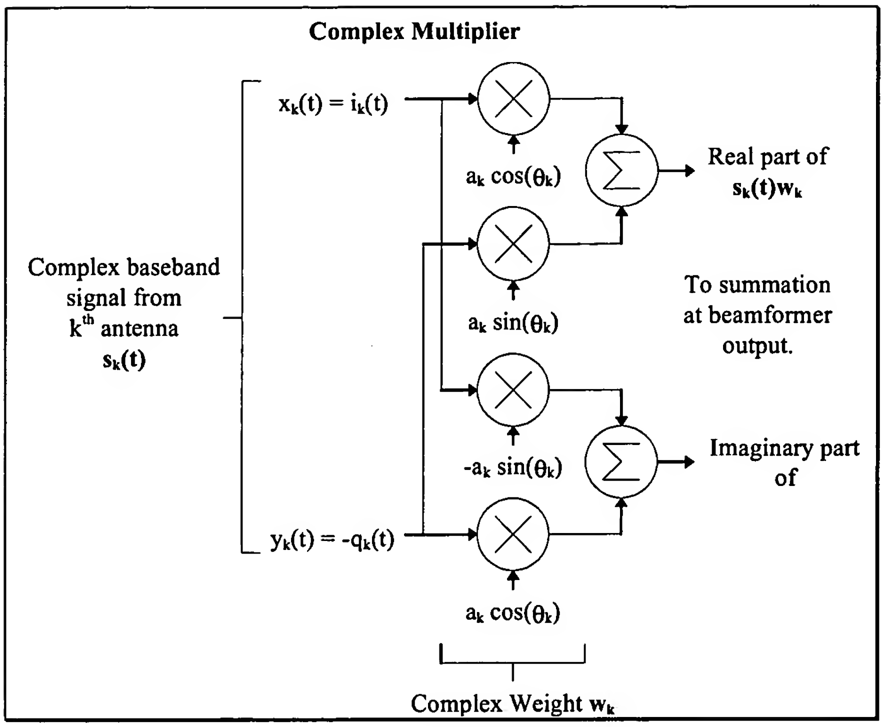
$$\mathbf{w_k = a_k\ e^{j\sin(\theta k)}}$$

$$\mathbf{w_k = a_k \cos(\theta_k) + j\ a_k \sin(\theta_k)}$$

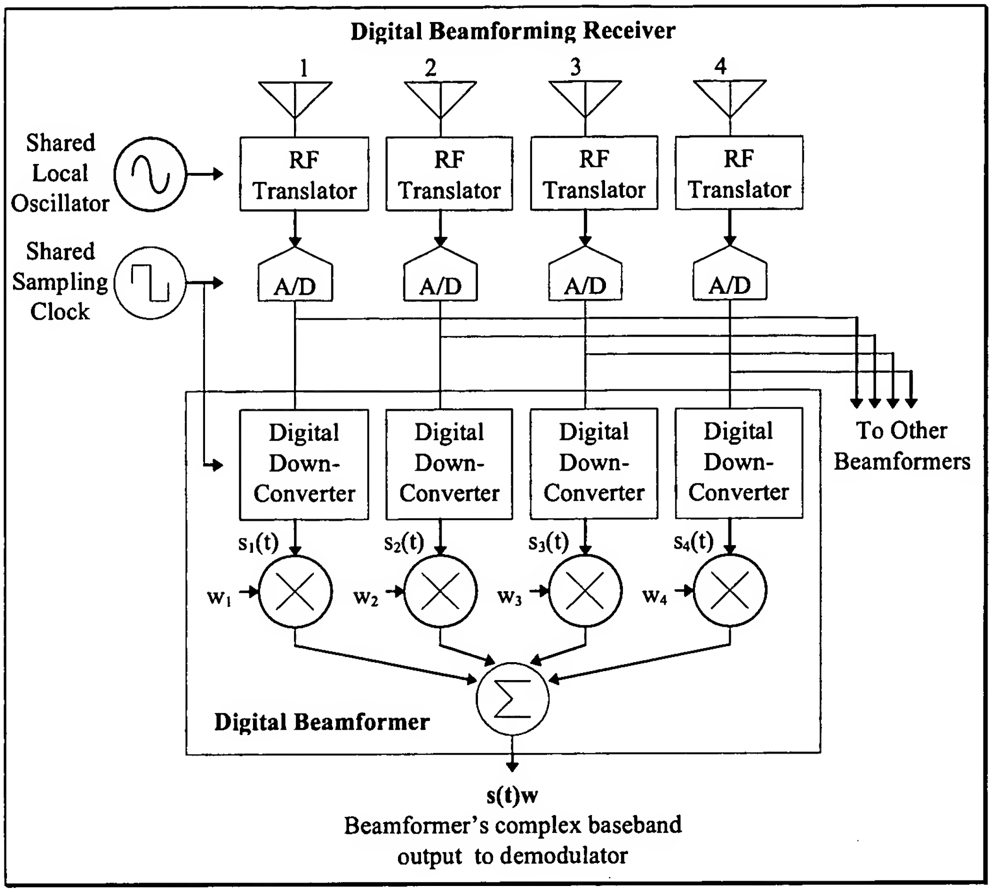
w_k is complex weight for the k^{th} antenna element
a_k is the relative amplitude of the weight
θ_k is the phase shift of the weight

A general-purpose DSP can implement the complex multiplication for each antenna element:

$$\mathbf{s_k(t)\ w_k = a_k\{\ [x_k(t) \cos(\theta_k) - y_k(t) \sin(\theta_k)] + j\ [x_k(t) \sin(\theta_k) + y_k(t) \cos(\theta_k)]\ \}}$$



The following figure shows a complete digital beamforming receiver. One set of antenna elements, RF translators, and A/D converters can be shared by a number of beamformers. All RF translators and A/D converters share common oscillators so that they all produce identical phase shifts of the signal. Within the digital beamformer, all digital down-converters share a common clock, are set for the same center frequency and bandwidth, and their digital local oscillators are in-phase so that all phase shifts are identical. Each DDC's baseband output is multipled by the complex weight for its antenna element, and the results are summed to produce one baseband signal with directional properties. A demodulator would then follow to recover information from the radio signal.



Adaptive Beamforming

The complex weights w_k for the antenna elements are carefully chosen to give the desired peaks and nulls in the radiation pattern of the antenna array. In a simple case, the weights may be chosen to give one central beam in some direction, as in a direction-finding application. The weights could then be slowly changed to steer the beam until maximum signal strength occurs and the direction to the signal source is found.

In beamforming for communications, the weights are chosen to give a radiation pattern that maximizes the quality of the received signal. Usually, a peak in the pattern is pointed to the signal source and nulls are created in the directions of interfering sources and signal reflections.

Adaptive Beamforming is the process of altering the complex weights on-the-fly to maximize the quality of the communication channel. Here are some commonly used methods:

> **Minimum Mean-Square Error** The shape of the desired received signal waveform is known by the receiver. Complex weights are adjusted to minimize the mean-square error between the beamformer output and the expected signal waveform.
>
> **Maximum Signal-to-Interference Ratio** Where the receiver can estimate the strengths of the desired signal and of an interfering signal, weights are adjusted to maximize the ratio.
>
> **Minimum Variance** When the signal shape and source direction are both known, chose the weights to minimize the noise on the beamformer output.

Often, constraints are placed on the adaptive beamformer so that the complex weights do not vary randomly in poor signal conditions. Some radio signals include "training sequences" so that an adaptive beamformer may quickly optimize its radiation pattern before the useful information is transmitted.

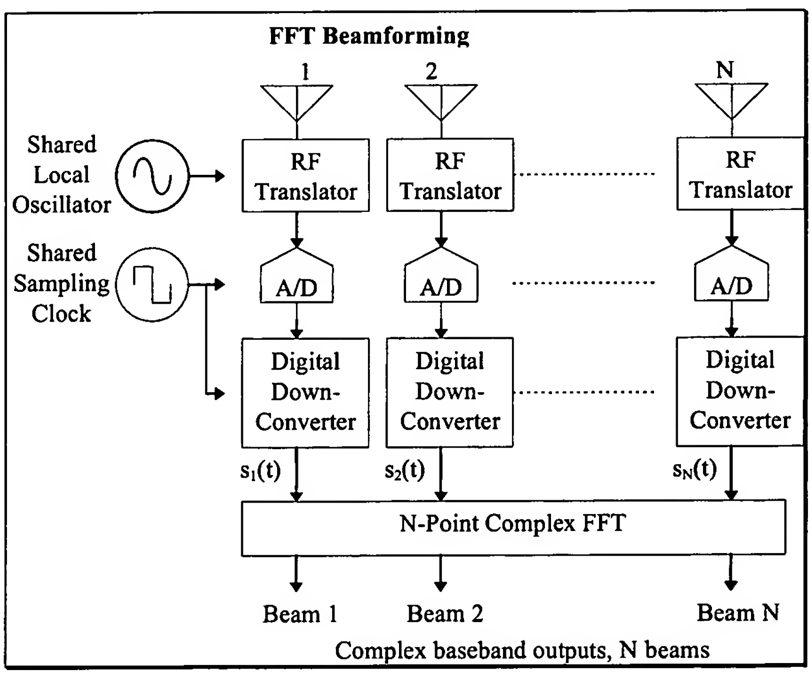
Smart Antennas

Adaptive beamforming systems for communications are sometimes referred to as "**smart antenna**" systems. For cellular telephone, one base station with a smart antenna system can support more than one user on the same frequency, as long as they are in different directions, by steering individual antenna beams at each user. This is sometimes called "**spatial domain multiple access**" (SDMA). It's estimated that the capacity of cellular telephone systems can be doubled by using smart antennas.

FFT's in Beamforming

In digital beamforming, many beamformers can share one set of antenna elements, rf translators, and A/D converters. The beamformers may have their central beams pointed in different directions. In situations where a fixed set of non-overlapping beams must be formed simultaneously (radar, sonar, direction-finding) an FFT can implement many beamformers efficiently.

The following figure shows an FFT beamformer with N antenna elements. Each element requires a digital down-converter. All DDC's produce a baseband sample simultaneously, and all of these are passed at once to an N-point complex FFT. The FFT then produces a set of N complex outputs, each of which is the next baseband sample for a different beam.



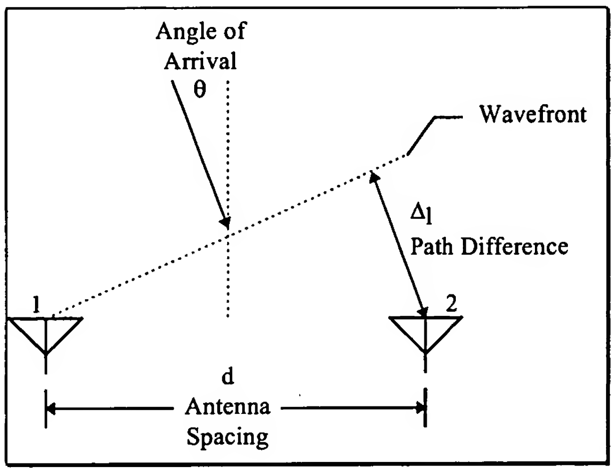
In this case, a "**spatial FFT**" is being performed: The FFT is processing a set of samples that are separated in space (not in time). Therefore, its outputs are a set of samples that are separated in direction (not in frequency).

FFT beamforming as shown above is not flexible. For a linear array, the N beams are fixed and equally spaced in direction. They range from -90 to +90 degrees from broadside of the array. The beams are orthogonal: the central peak of any beam lies in a null on all other beams. Such a set of beams is useful for radar mapping, but not very useful for communications.

It is possible to use FFT's for beamforming in communications. A set of FFT outputs can be combined, using complex weights and sums as before, to form arbitrary radiation patterns. This is called "**beam-space beamforming.**" The previous approach of combining baseband signals from different antenna elements is called "**element-space beamforming.**"

Super-Resolution Direction Finding

The term "**super-resolution**" applies to the ability to measure the angle of arrival of a radio signal with much higher resolution than the beam width of the antenna array. The method requires accurately measuring the phases of the signals from the array elements and, from these, calculating the angle of arrival.



A wavefront from direction θ arrives at antenna 1 first. Then, after travelling an additional path distance Δl it arrives at antenna 2.

$$\Delta l = d \sin\theta$$

The pah difference results in a phase difference Δϕ between the signals from the two antennas:

$$\Delta\phi = 2\pi\, \Delta l / \lambda$$

$$\Delta\phi = 2\pi\, d \sin\theta / \lambda$$

A direction-finding system calculates the angle of arrival from the phase difference:

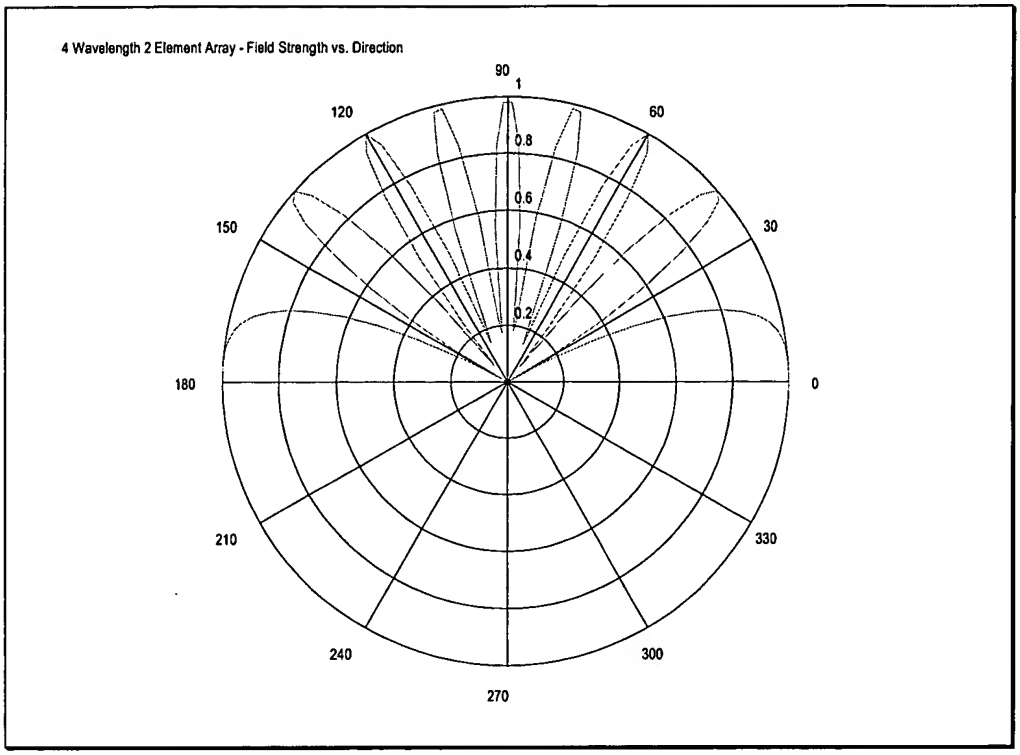
$$\theta = \sin^{-1} (\Delta\phi\, \lambda / (2\pi\, d))$$

For a super-resolution result to be accurate, the arriving wave must be a direct signal from the source - a "**plane wave**" with a straight wavefront. Signal reflections (multipath) and interfering signals cause super-resolution systems to fail. A super-resolution system cannot operate if two or more signal sources share the same frequency, since the receiver's output phase no longer reflects the phase of an incoming plane wave.

Beamforming can be used for direction finding by rotating the central beam of an array to give maximum received signal strength. With this method, the angular resolution is limited by the beam width produced by the beamformer. Also, false measurements will occur if a side lobe is mistakenly steered to the signal source, instead of the array's central lobe. However, it is possible to measure the directions of multiple sources and to identify the directions of reflections with a beamforming system.

For two antenna elements spaced at 4 wavelengths, the following diagram shows the radiation pattern that a beamformer would produce. The main drawback of the beamforming approach - many side lobes unless

many antenna elements are used - is apparent. A super-resolution system using the same two antennas could measure direction accurately, provided that the only an undistorted plane wave is arriving.



References

John Litva and Titus Kwok-Yeung Lo, Digital Beamforming in Wireless Communications, Artech House, Norwood, MA, 1996.

Warren L. Stutzman and Gary A. Thiele, Antenna Theory and Design, John Wiley & Sons, New York, 1981.

Advanced Antenna Systems in Wireless Communications

- Introduction.
- Multiple antenna systems and terminology.
- Antenna arrays in wireless communications.
- Beamforming systems.
- Direction Of Arrival (DOA) estimation methods.
- Summary.

Multiple Input Multiple Output (MIMO) Channels

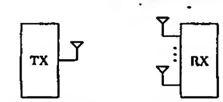
- The term MIMO can be used as a generic term to cover the following:
  - Beamforming.
  - Transmit/receive diversity and space time coding.
  - Spatial multiplexing.
- Multiple antennas are used to increase system throughput and immunity to fading, interference, and noise.
- Space-time coding and beamforming with space-time algorithms for signal transmission and detection at the receiver.
- Multiple antenna techniques are already used in practical communication systems (for example, at base station), and are embedded in current standards as UMTS and IEEE 802.16 (WiMax).

Advantages of MIMO Systems

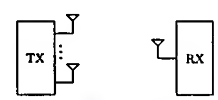
- *Multiplexing* where different signals sent through different antennas $\rightsquigarrow$ increase in transmission data rate.
- *Diversity* where the same signal is sent through different antennas $\rightsquigarrow$ better performance.
- There is always a tradeoff between multiplexing and diversity gain:
  - High diversity is achieved at low data rate.

Terminology

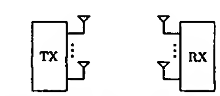
SISO (single-input single-output)



SIMO (single-input multiple-output)

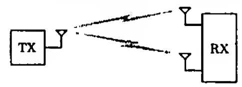


MISO (multiple-input single-output)



MIMO (multiple-input multiple-output)

- Consider the case of one transmit antenna and two receive antennas



- For a flat fading channel, the received signals at the antennas are:

$$\begin{aligned} y_1 &= h_1 s + z_1 \\ y_2 &= h_2 s + z_2 \end{aligned} \qquad \Rightarrow \boldsymbol{y} = s\boldsymbol{h} + \boldsymbol{z}$$

h_1, h_2 channel gains
z_1, z_2 mutually uncorrelated noises of variance N_0.
$\boldsymbol{y} = [y_1, y_2]^T$, $\boldsymbol{h} = [h_1, h_2]^T$, $\boldsymbol{z} = [z_1, z_2]^T$

- To recover the symbol s we combine y_1 and y_2 as

$$\begin{aligned} \hat{s} &= w_1^* y_1 + w_2^* y_2 = s(w_1^* h_1 + w_2^* h_2) + w_1^* z_1 + w_2^* z_2 \\ &= \boldsymbol{w}^H \boldsymbol{y} = s\boldsymbol{w}^H \boldsymbol{h} + \boldsymbol{w}^H \boldsymbol{z} \end{aligned}$$

- The combining coefficients that maximizes the SNR of the combined signal are

$$w_1 = h_1, \qquad w_2 = h_2$$

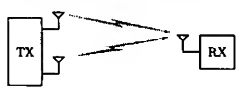
This is known as Maximum Ratio Combining (MRC).

- The SNR of the combined signal becomes:

$$\Gamma = \frac{\left(|h_1|^2 + |h_2|^2\right) E_s}{N_0}$$

- When h_1 and h_2 are uncorrelated, a receive diversity gain of order 2 is obtained.
- Note that channel state information is needed at the receiver for this combining scheme.

- Consider the case of two transmit antenna and one receive antenna



- Let the symbol s be transmitted from each antenna
- Multiplying s by w_1^* at antenna 1 and by w_2^* at antenna 2, the received signal is

$$y = w_1^* h_1 \frac{s}{\sqrt{2}} + w_2^* h_2 \frac{s}{\sqrt{2}} + z = \frac{s}{\sqrt{2}} \boldsymbol{w}^H \boldsymbol{h} + z$$

- The coefficients that maximizes the received SNR are

$$w_1 = \frac{h_1^*}{\sqrt{|h_1|^2 + |h_2|^2}}, \qquad w_2 = \frac{h_2^*}{\sqrt{|h_1|^2 + |h_2|^2}}$$

  - Channel state information is needed at the transmitter to generate the optimum combining coefficients.

- The received SNR with the optimum coefficients is given by

$$\Gamma = \frac{\left(|h_1|^2 + |h_2|^2\right) E_s}{2N_0}$$

  - We have transmit diversity of order 2.
  - This diversity requires the channel knowledge at the transmitter!
  - More complex than receive diversity.
- If the channels $\{h_1, h_2\}$ are not known at the receiver and $\{w_1, w_2\}$ are fixed and independent of $\{h_1, h_2\}$, it is impossible to have transmit diversity.

Antenna Arrays

- The performance of a multiple antennas system depends on the relative placement of the antenna elements with respect to each other
  - Closely spaced antennas (e.g. $\lambda/2$) provide both angle and polarization diversity.
  - Widely spaced antennas (e.g. λ) provide both spatial and polarization diversity.
- In general, the array factor (diagram) of a multiple antennas system is given by

$$A(\theta,\phi) = \boldsymbol{w}^H \boldsymbol{a}(\theta,\phi) .$$

$$\boldsymbol{w} = [w_0, w_1, \cdots, w_{N-1}]$$
$$\boldsymbol{a}(\theta,\phi) = \left[e^{j\frac{2\pi}{\lambda}\delta_0}, e^{j\frac{2\pi}{\lambda}\delta_1}, \cdots, e^{j\frac{2\pi}{\lambda}\delta_{N-1}}\right], \qquad \delta_n = x_n\cos(\phi)\sin(\theta) + y_n\sin(\phi)\sin(\theta) + z_n\cos(\theta)$$

(x_n, y_n, z_n) are the cartesian coordinates of Antenna n.
$\boldsymbol{w}$ is the weight vector.
$\boldsymbol{a}(\theta,\phi)$ is the steering vector (or space vector or array response vector).
ϕ denotes the azimuth angle.
θ denotes the zenith angle.

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Array Antenna Geometry

- The geometry of an antenna array controls its steering vector (space vector) $\boldsymbol{a}(\theta,\phi)$.
- The Geometrical configuration can be linear, circular, planar, cylindrical, spherical, or other.

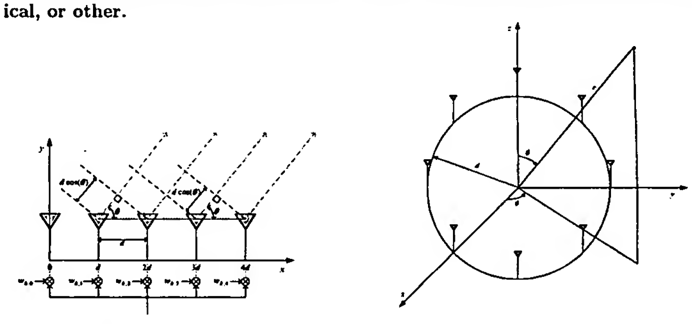

Figure 1: A Uniform Linear Array (ULA) and a Uniform Circular Array (UCA).

– Uniform Linear Array: $\delta_n = nd\cos(\theta))$
– Uniform Circular Array: $\delta_n = nd\sin(\theta)\cos(\phi - 2\pi n/N)$

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Antenna Array Design Methods

- Depending on the application, the antenna array diagram (pattern) should fulfill some requirements
  - Directivity.
  - Beamwidth of the antenna array.
  - The side lobe levels of the diagram.
  - etc ...
- The array diagram can be controlled in many different ways using the following controls:
  - The geometrical configuration of the array.
  - The antenna diagram of each Antenna Element (AE).
  - The relative displacement between the AEs.
  - The weight applied on each AE.
- In practice, the last control is the easiest to change and is mostly used to control the array diagram.
  - This control is referred to as *beamforming* in wireless communication.

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Concept of Beamforming

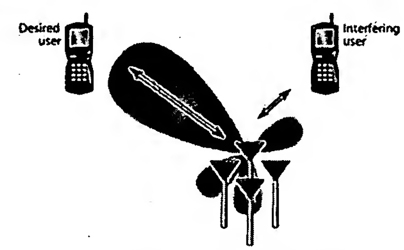
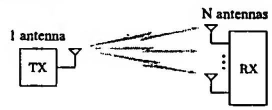


Figure 2: A beamforming smart antennas system.

- The process of combining the signals from different antenna elements is known as *beamforming*.
- Beamforming is a spatial filtering process that separates the desired signal from interfering signals.
- The main objective is to detect and estimate the signal of interest in the presence of interference.

- Let us consider a SIMO system



- The key idea is to use the proper weights at the receiver

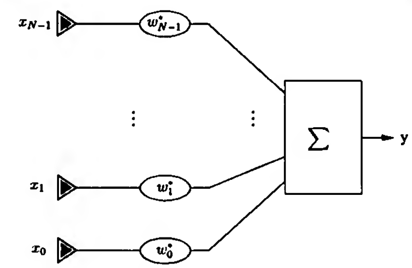
$$y = \sum_{i=0}^{N-1} w_i^* x_i \;=\; \boldsymbol{w}^H \boldsymbol{x}$$

$\boldsymbol{w} = [w_0, w_1, \cdots, w_{N-1}]^T$
$\boldsymbol{x} = [x_0, x_1, \cdots, x_{N-1}]^T$

- The received vector, x, contains the desired signal, external interference, and noise

$$\begin{aligned} \boldsymbol{x} &= \boldsymbol{u} + \boldsymbol{x}_i + \boldsymbol{z} \\ &= \boldsymbol{u} + \boldsymbol{n} \end{aligned}$$

$\boldsymbol{u}$ is the desired signal.
$\boldsymbol{n}$ is the total noise.



- There are two types of beamforming:
  - Conventional Beamforming (Fixed-beams): The coefficients do not depend on the input/output array signals.
  - Adaptive Beamforming: The coefficients are determined and optimized based on the input/output array signals.

- Let us consider a MISO system

- The key idea is use the proper weights at the transmitter to transmit the desired signal to the receiver and reduce interference.
- Assuming that the same symbol x is transmitted from all antenna elements, the received signal is

$$y = x \sum_{i=0}^{N-1} w_i^* h_i \;=\; s\boldsymbol{w}^H \boldsymbol{h}$$

$\boldsymbol{w} = [w_0, w_1, \cdots, w_{N-1}]^T$
$\boldsymbol{h} = [h_0, h_1, \cdots, h_{N-1}]^T$ is the channel vector.

- Channel state information is needed at the transmitter to properly choose the weight vector.

- The concept of fixed-beams can be defined as a set of beams that cover a specific area.
- Typically each beam serves more than oner user.
- The FB weight vectors are generated by a Beam Forming Network (BFN) which consists of an $N \times N$ BFN Matrix

$$\boldsymbol{T} = [\boldsymbol{w}_0, \boldsymbol{w}_1, \cdots, \boldsymbol{w}_{N-2}, \boldsymbol{w}_{N-2}]$$

  - The weight vector of the nth beam can be written as

$$\boldsymbol{w}_n = [w_0, w_1, \cdots . w_{N-2}, w_{N-1}]^T$$

- A conventional multi-beam system must be calibrated in order to ensure similar amplitude differences between UP and DL signal paths for each beam.
- Beamforming can be done in two different ways:
  - Beamforming in RF (analog) using analog BFN.
  - Beamforming in baseband (digital) using a digital signal processor.

Conventional Fixed-Beams systems

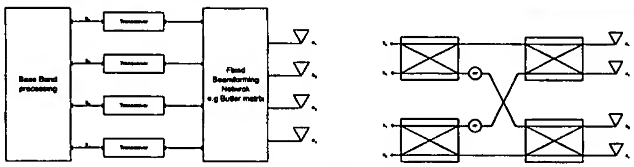


Figure 3: Analog beamforming network.

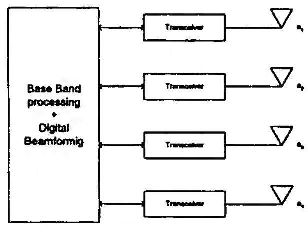


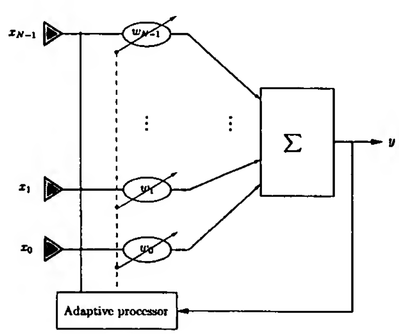
Figure 4: Digital beamforming network.

Conventional Fixed-Beams systems

- Advantages:
  - Fixed multi-beams systems are easy to implement and are widely available and cheap.
  - Fixed multi-beams systems do not require a lot of interaction with the base station receiver.
- Disadvantages:
  - Fixed multi-beams systems are not able to take advantage of path diversity.
  - Fixed multi-beams systems are not able to attenuate or eliminate interferers that arrive with direction of arrival close to the desired signal.

Adaptive Beamforming

- In adaptive beamforming, the weights of the antennas are adapted according to a certain objective function.
- The objective function can be:
  - Maximizing the received antenna gain or array factor.
  - Maximizing the received signal-to-noise ratio.
  - Etc ...



Criteria of Adaptive Beamforming

- Minimum of the Mean-Square Error (MSE)
  - The weights are chosen to minimize the following cost function

$$J(w) = E\left\{|d-\hat{d}|^2\right\} = E\left\{|d - w^H x|^2\right\}$$

d is a known reference signal.
x is the received vector.

  - The optimal MMSE solution is give by

$$w = R^{-1}E\{xd^*\}$$

$R = E\{xx^H\}$ is the covariance matrix of the received vector.

  - The optimal solution requires a matrix inversion which, depending on the number of AEs, can be difficult to compute.
    - Adaptive updating of the different weights can reduce this complexity

$$w^{(k+1)} = w^{(k)} + \Delta(d-\hat{d})x^*$$

Criteria of Adaptive Beamforming

- Maximum Signal-to-Noise Ratio (SNR):
  - The Maximum SNR algorithm chooses the weights such that the received SNR is maximized

$$J(w) = \frac{E\{|w^H u|^2\}}{E\{|w^H n|^2\}} = \frac{w^H R_u w}{w^H R_n w}$$

  - Optimization can be formulated as

$$\min_{w} \left(w^H R_n w\right) \qquad \text{subject to} \qquad w^H R_u w = 1$$

  - The optimum Maximum SNR solution is given by

$$w_{\text{opt}} = \mathcal{P}\left\{R_n^{-1} R_u\right\}$$

  $\mathcal{P}\{\cdot\}$ denotes the principle eigenvector of a matrix.
    - Requires matrix inversion.
    - Requires knowledge of the Direction of Arrival (DOA) of the desired signal.
    - Requires knowledge about R_n (noise statistics)!

Direction Of Arrival (DOA) Estimation

- The purpose of direction finding is to estimate the number of sources and their DOA's
- Many beamforming algorithms require information about the DOA.
- The array model is given by

$$x = a(\theta)s + n$$

  s is the source.
  $a(\theta)$ is the steering vector of the source.
  θ is the unknown DOA of the source to be estimated.
- Example: In the Uniform Linear Array (ULA) case the steering vector is given by

$$a(\theta) = \left[1, e^{j(2\pi/\lambda)d\sin\theta}, \cdots, e^{j(2\pi/\lambda)d(N-1)\sin\theta}\right]^T$$

- Our objective to estimate the direction of arrival θ of the source s

$$\mathcal{M} = \{a(\theta); \theta \in \Theta\}$$

  Θ is the so-called Field-Of-View (FOV).

Conventional Beamformer

- A conventional beamformer scans the beams to evaluate the received power in each direction and to find the signal DOA's from the maxima of the array output

$$\begin{aligned} P_{\text{CBF}}(\theta) &= E\{|a^H(\theta)x|^2\} \\ &= a^H(\theta)Ra(\theta) \end{aligned}$$

  $R = E\{xx^H\}$.
- The output power as a function of the angle of arrival can be estimated from the autocorrelation matrix R and the steering vector $a(\theta)$.
- Sample version of conventional beamformer:

$$P_{\text{CBF}}(\theta) = a^H(\theta)\hat{R}a(\theta)$$

- This can be done in discrete steps of the angular region until the direction is found.

Capon Beamformer Method

- Consider the spatial filter (beamformer) with the output

$$y = w^H x$$

  x is the input of the beamformer.
  w is the weight vector of the beamformer.
- The output power is given by

$$\begin{aligned} E\{|y|^2\} &= E\{|w^H x|^2\} = w^H E\{xx^H\}w \\ &= w^H R w \end{aligned}$$

- Capon method consists of steering the array towards a particular DOA θ and try to reject the signals at all remaining directions

$$\min_{w} E\{|y|^2\} \qquad \text{subject to} \qquad w^H a(\theta) = 1$$

  The problem can be rewritten as

$$\min_{w} w^H R w \qquad \text{subject to} \qquad w^H a(\theta) = 1$$

- The optimum solution (output power)

$$P_{\text{CAPON}} = \frac{1}{a^H(\theta)R^{-1}a(\theta)}$$

Summary

- Advanced antenna systems are becoming an integral part of future wireless communications.
- Advanced antenna systems can increase the throughput of wireless communication systems.
- Advanced antenna systems provides a good and flexible compromise between multiplexing and diversity gain.

Issue:

The Versatile Butler Matrix

Use of a two-dimensional Butler matrix beam-forming network to produce a pincushion footprint display for mobile satellite system coverage

by Bruno Pattan, Federal Communications Commission

Sun, November 14. 2004

Both the IRIDIUM (LEO) and Odyssey (MEO) mobile satellite systems use constellations to provide worldwide continuous coverage. Each satellite in the constellation produces antenna beams covering wide areas. However, because the coverage areas are large, and the users in a coverage area may exceed the capacity of a single beam, the coverage is blanketed by multiple spot beams, which approximate a cellular structure on the ground. This, like in terrestrial cellular systems, permits frequency reuse, thus enhancing the capacity of the systems. Additional means are used to enhance capacity, of course, but this note will confine its attention to the cell topology only. The benefits accrued from using cellular partitioning with spot beams also result from the fact that the ground transceiver will see a higher effective isotropic radiated power (EIRP) emanating from the satellite and will see a higher G/T (receive antenna gain vs. noise temperature) when in its transmission mode.

The spot beams are generated by an on-board multi-beam antenna subsystem that lays down multiple spot beams approximating a cellular structure on the surface of the Earth. The multi-beam antenna design used by both Motorola and TRW use a two-dimensional Butler matrix beam-forming network to produce a pincushion footprint display within each coverage area.

The Butler matrix has been used extensively over the years in radar and electronic warfare (electronic support measures) and satellite systems.[3] The Butler matrix consists of passive four-port hybrid power dividers and fixed phase shifters. It has N input ports and N output ports. As a beam-forming network, it is used to drive an array of N antenna elements. It can produce N orthogonally space beams overlapping at the –3.9 dB level, and having the full gain of the array. The network is most commonly used to produce volumetric beams in a pincushion deployment, where to be orthogonal sinX/X patterns must be spaced so that the cross-over is at about 4 dB down ($2/\pi$), and the sidelobes will be down 13.2 dB. Tapering will violate this requirement. For a cosine taper, for example, the cross-over level becomes approximately 9.5 dB for orthogonality.

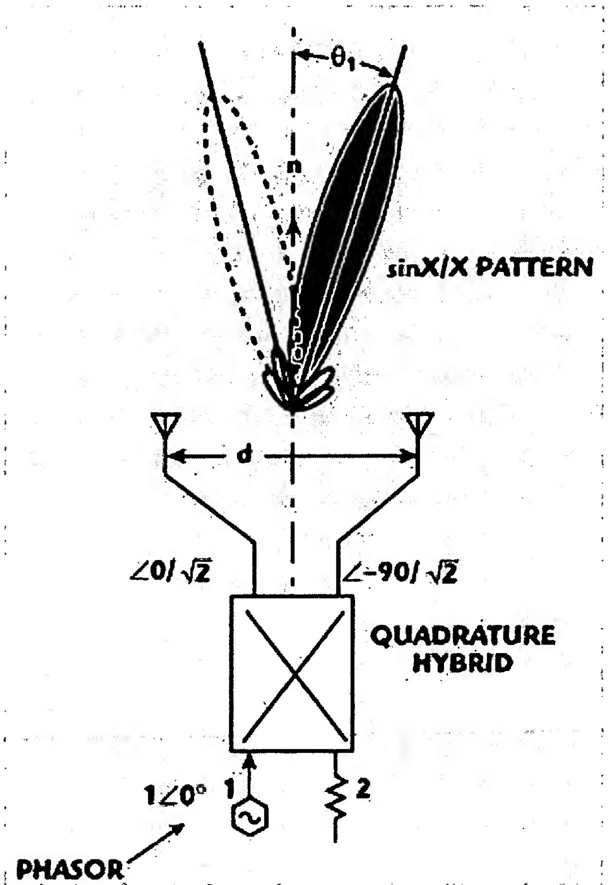


Fig. 1 A two-beam Butler matrix.

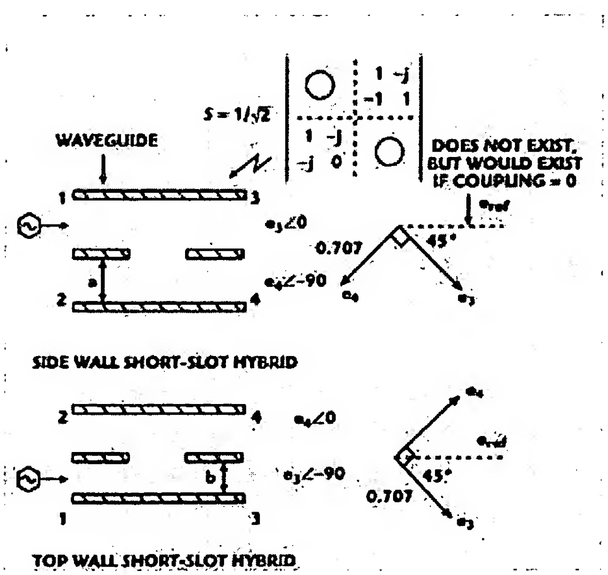
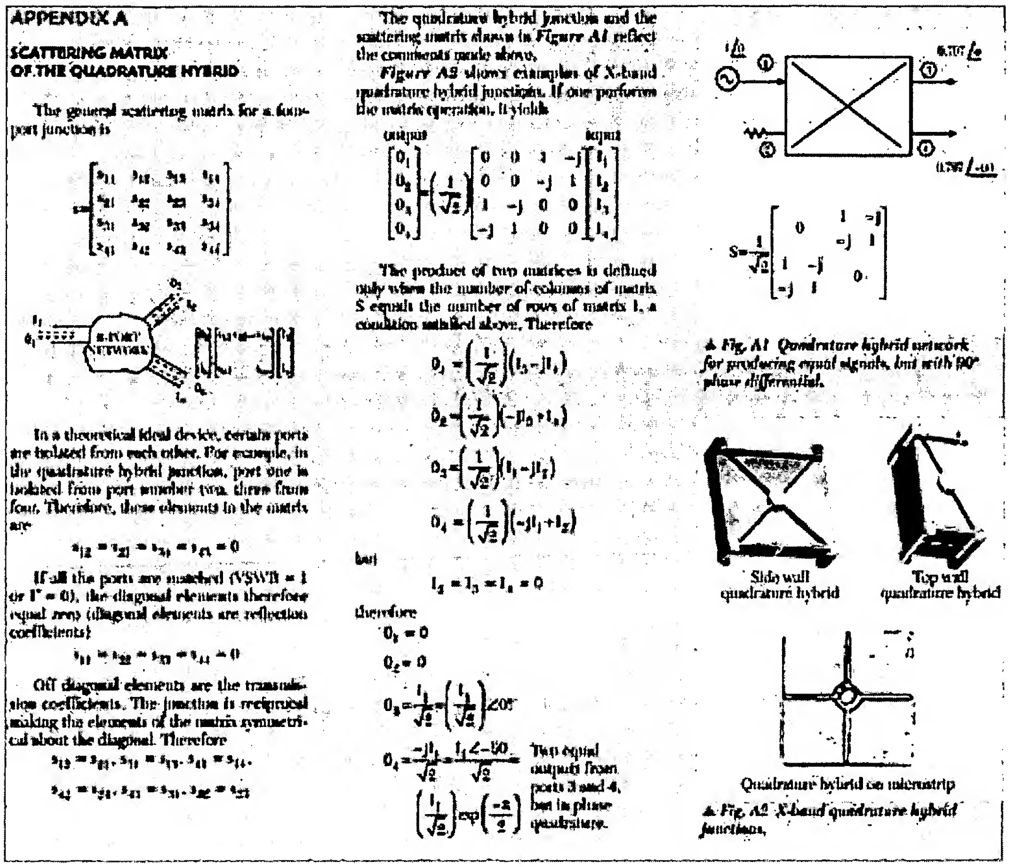


Fig. 2 Phase relationships in riblet short-slot hybrids, the basic building block of the Butler matrix.

A generic version of the Butler matrix used as a beam-forming network is shown in *Figure 1*. It consists of a 3 dB quadrature hybrid[1] (see *Figure 2* and *Appendix A*) driving two antenna elements with separation d. Note the amplitude and phase relationships for the hybrid structure.



APPENDIX A

SCATTERING MATRIX OF THE QUADRATURE HYBRID

The general scattering matrix for a four-port junction is

$$S = \begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} \\ s_{21} & s_{22} & s_{23} & s_{24} \\ s_{31} & s_{32} & s_{33} & s_{34} \\ s_{41} & s_{42} & s_{43} & s_{44} \end{bmatrix}$$

In a theoretical ideal device, certain ports are isolated from each other. For example, in the quadrature hybrid junction, port one is isolated from port number two, three from four. Therefore, these elements in the matrix are

$$s_{12} = s_{21} = s_{34} = s_{43} = 0$$

If all the ports are matched (VSWR = 1 or $\Gamma = 0$), the diagonal elements therefore equal zero (diagonal elements are reflection coefficients)

$$s_{11} = s_{22} = s_{33} = s_{44} = 0$$

Off diagonal elements are the transmission coefficients. The junction is reciprocal making the elements of the matrix symmetrical about the diagonal. Therefore

$$s_{13} = s_{31}, s_{14} = s_{41}, s_{41} = s_{14},$$

$$s_{42} = s_{24}, s_{43} = s_{34}, s_{32} = s_{23}$$

The quadrature hybrid junction and the scattering matrix shown in *Figure A1* reflect the comments made above.

Figure A2 shows examples of X-band quadrature hybrid junctions. If one performs the matrix operation, it yields

$$\begin{array}{c} \text{output} \\ \begin{bmatrix} O_1 \\ O_2 \\ O_3 \\ O_4 \end{bmatrix} \end{array} = \left(\frac{1}{\sqrt{2}}\right) \begin{bmatrix} 0 & 0 & 1 & -j \\ 0 & 0 & -j & 1 \\ 1 & -j & 0 & 0 \\ -j & 1 & 0 & 0 \end{bmatrix} \begin{array}{c} \text{input} \\ \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} \end{array}$$

The product of two matrices is defined only when the number of columns of matrix S equals the number of rows of matrix I, a condition satisfied above. Therefore

$$O_1 = \left(\frac{1}{\sqrt{2}}\right)(I_3 - jI_4)$$

$$O_2 = \left(\frac{1}{\sqrt{2}}\right)(-jI_3 + I_4)$$

$$O_3 = \left(\frac{1}{\sqrt{2}}\right)(I_1 - jI_2)$$

$$O_4 = \left(\frac{1}{\sqrt{2}}\right)(-jI_1 + I_2)$$

but

$$I_2 = I_3 = I_4 = 0$$

therefore

$$O_1 = 0$$

$$O_2 = 0$$

$$O_3 = \frac{I_1}{\sqrt{2}} = \left(\frac{I_1}{\sqrt{2}}\right)\angle 0°$$

$$O_4 = \frac{-jI_1}{\sqrt{2}} = \frac{I_1\angle -90}{\sqrt{2}} = \left(\frac{I_1}{\sqrt{2}}\right)\exp\left(\frac{-\pi}{2}\right)$$

Two equal outputs from ports 3 and 4, but in phase quadrature.

$$S = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 1 & -j \\ 0 & 0 & -j & 1 \\ 1 & -j & 0 & 0 \\ -j & 1 & 0 & 0 \end{bmatrix}$$

▲ *Fig. A1 Quadrature hybrid network for producing equal signals, but with 90° phase differential.*

▲ *Fig. A2 X-band quadrature hybrid junctions.*

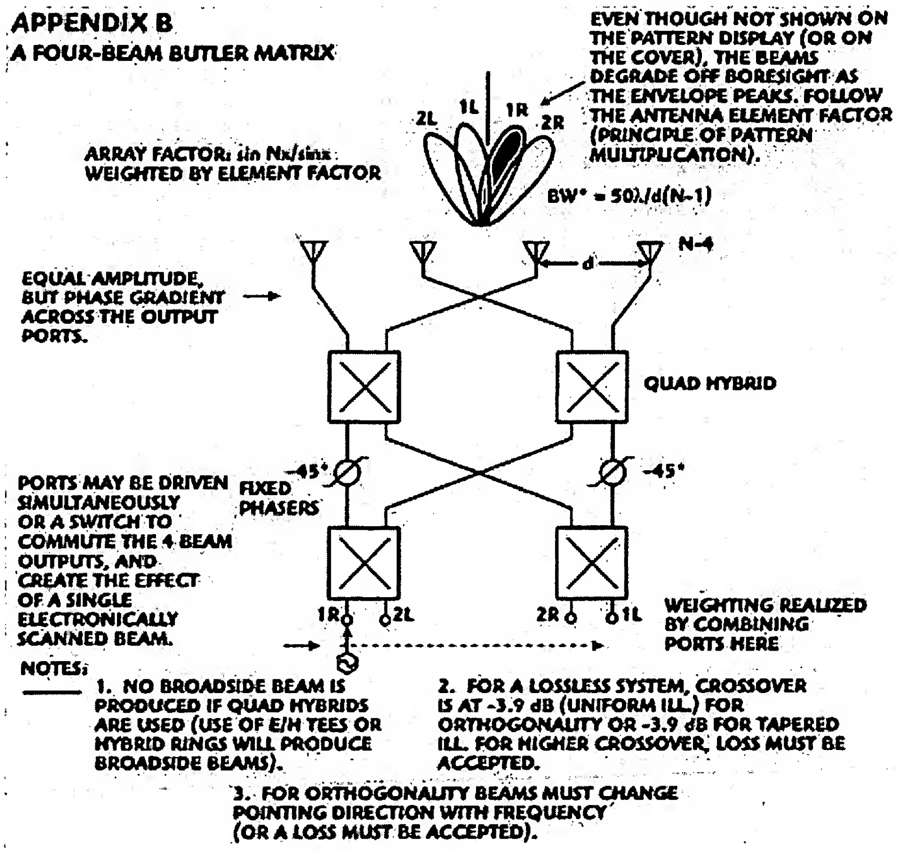
For example, feeding the lower left-hand port of the coupler in the Butler matrix results in the antenna array being uniformly illuminated and differential phased to point the resulting beam peak to the right of boresight in direction O. With the phasing indicated, the relationship becomes

$$(2\pi\, d/\lambda) \sin\theta_1 - \pi/2 = O$$

$$\text{or } \sin\theta = \lambda/4d$$

Feeding the lower right port 2 results in a beam pointing to the left of boresight at an angle $\theta_2 = -\lambda/4d$. If both ports one and two are driven, two beams will be produced at angles $\theta = \pm\lambda/4d$. The matrix produces N (equal 2 here) orthogonally spaced beams overlapping at the -3.9 level.

A slightly more detailed version producing four beams is shown in *Appendix B*. This consists of several quadrature hybrids and fixed phase shifters. It is noted that if one traces a phasor through the network, no boresight beam is formed and the beams are symmetrically deployed about the array axis. For the port driven in the figures it can be observed that the phase front across the aperture elements is -45°, -90°, -135° and -180°. Therefore, the shaded beam is produced.



The four-beam matrix may also have a sequencing switch on the input ports to scan to any one of the four positions shown. A more elaborate labyrinth can scan or produce many more beams.

In the basic array, the number of beams is equal to the number of elements, and the array factor is of the form sinNx/sinx, where N is the number of elements. Kraus[10] has shown that the magnitude of the field intensity in the far field of a linear array of N istropic radiators is given by

$$E(\theta) = E_e(\theta)\frac{1}{N}\frac{\sin(N\phi/2)}{\sin(\phi/2)}$$

where

E (θ) = element factor (weighs the array factor)

$\theta = (2\pi d/\lambda)\sin\theta - \delta$

δ = progressive phase difference generated by the matrix and is equal to

$\delta_k = (2k - 1)\pi/N$,

k = 1,2, ..., N/2

k = beam number

Note in the two-element array previously shown

$$\delta_k = (2k\text{-}1)\,\pi/N$$

$$\delta = \pi/N = \pi$$

The location of the beams can be found from the following relationship

$$\sin\theta = (\pi/Nd)\,[k-(1/2)]$$

The first sidelobe is down 13.5 dB, which is typical for a linear array with equal amplitudes and equal spacing. Interestingly, the beams cross over at the 3.9 dB points, which suggest that the beams are orthogonal and the network is lossless.

The sidelobes can be significantly improved by coherently combining two output ports to give a cosine variation with the sidelobes down by 23 dB, but with a beam distention of 35 percent. *Figure 3* illustrates the addition of two beams each having uniform illumination (with sin x/x far field) to obtain a cosine illumination. In general, n+1 beams can be added to form a $(\text{consine})^n$ illumination.

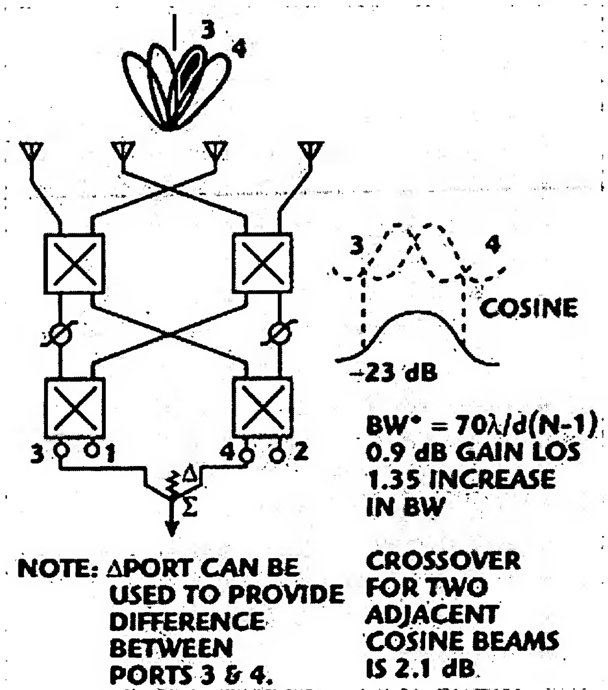


Fig. 3 Cosine illumination achieved by combining two adjacent beams.

There are other tricks that can be manifested by the matrix. The beams can be made to have limited scan by the use of the network shown in *Figure 4*. Power is varied to ports 3 and 4 (for example) and the variable power divider controls the power to each port. Varying the division of power steers the beam between individual beam axes.

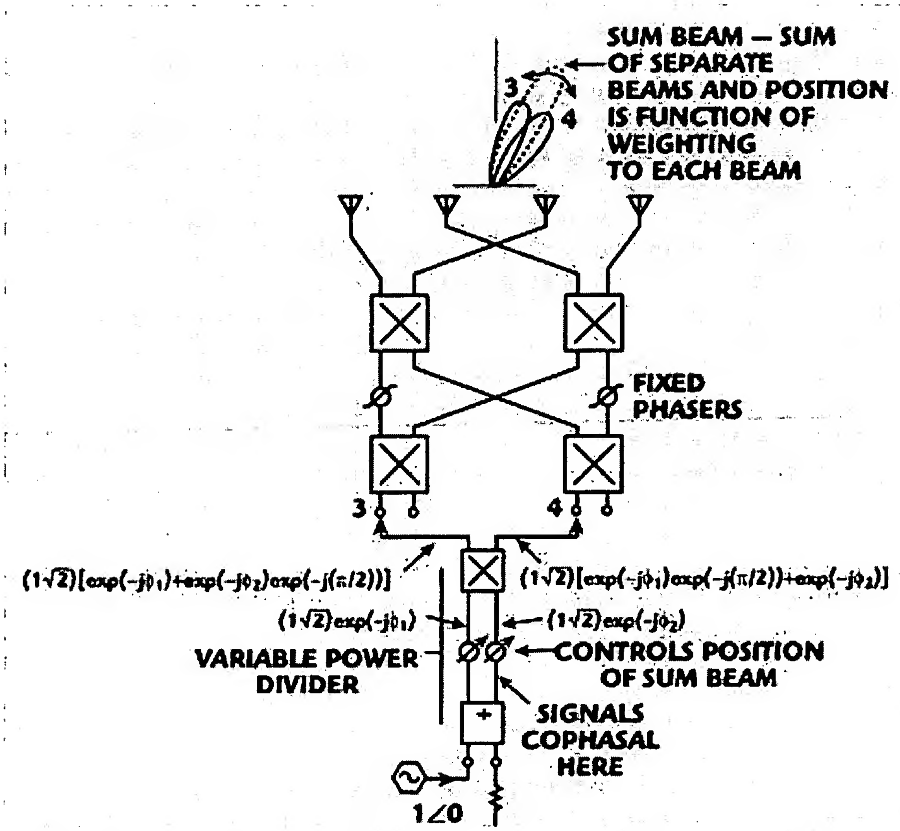


Fig. 4 Beam scanning in a Butler matrix using a variable power divider.

Multiple volumetric beams can be generated by a Butler matrix by dividing the array into rows and columns. This is depicted in *Figure 5*. The columns consist of linear Butler matrices where the outputs of each column are the vertical number of beams. For this example, four beams are in each vertical stack. The outputs of the column matrices drive four row matrices. These are also Butler matrices. The outputs of the row matrix are used to produce four squinted vertical stacks. It is noted that prior to squinting, the column matrices produce vertical beam stacks (4 per stack) that are all pointing in the same direction.

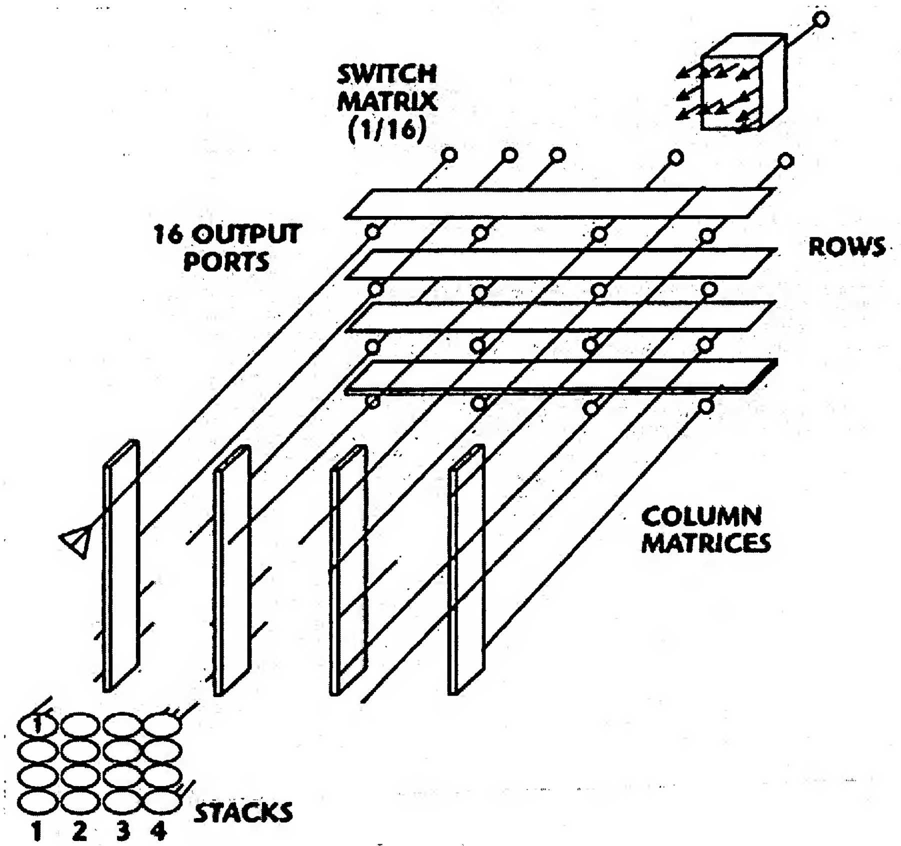


Fig. 5 A two-dimensional Butler matrix producing orthogonal beams.

For example, beam 1 is formed by column matrix 1, but is squinted upward from the horizontal boresight plane by the inherent operations of the matrix. The beam is now squinted to the left from the vertical boresight plane by the phasing of the row matrix 1 and its port 2. Correlating Appendix A with Figure 5 will help to make this clear.

There is a practical problem associated with the system shown in Figure 5. The stacked beam will form pencil beams only if the aperture size in the direction perpendicular to the column matrices is comparable to the array lengths. If there are horns with a cosine distribution, the aperture width in that direction would have to be equal to $w = 69\ \lambda/BW$, where BW is the beamwidth in the vertical direction. The beamwidth in the vertical direction results from the array length. In the orthogonal direction, a very large flare may be required.

Generation of beams for volumetric coverage by a Butler matrix labyrinth requires considerable circuit complexity. Additional complexity results from circuitry to reduce the antenna beam sidelobe levels. With attendant low cross-over (9.5 dB for cosine taper and lower still for a smaller sidelobe level or high loss with a higher cross-over), the Butler matrix requires

$$H = (N/2) \log_2 N$$

$$= (N/2)\,(\log_{10} N / \log_{10} 2)$$

$$= 3.32\,(N/2) \log_{10} N$$

quadrature couplers, plus fixed phase shifters. The number of fixed phase shifters is

$$P = (N/2)\cdot$$

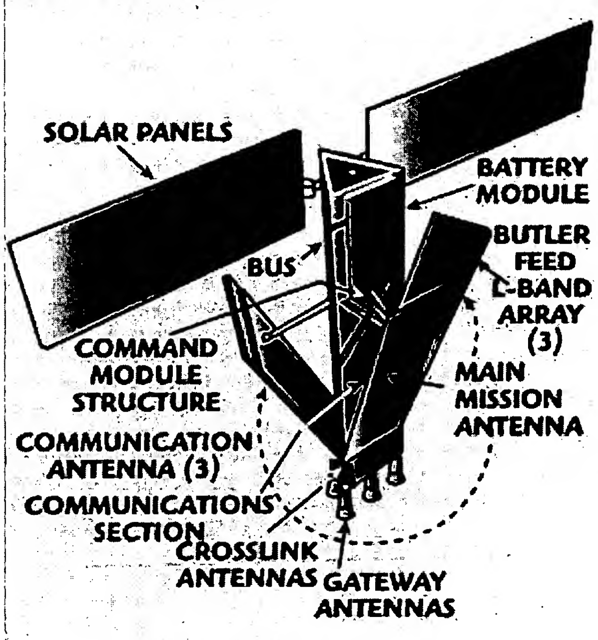
$$(\log_2 N - 1)$$

For example, for a 64-beam system (8 x 8), 16 planar matrix feeds are required with 12 hybrid junctions and 16 fixed phasers for each matrix.

A sketch of the IRIDIUM spacecraft using the Butler matrix beam-forming network is shown in *Appendix C*. Each of the three communications signal arrays generate 16 spot beams. These beams are juxtaposed to produce 48 beams in the coverage area.

APPENDIX C

SKETCH OF THE IRIDIUM SATELLITE DESIGN



COMMUNICATION ANTENNA (3):

- 86 cm WIDE
- 186 cm HIGH
- 4 cm THICK
- 106 RADIATING ELEMENTS
- 16 BEAMS PER ANTENNA
- 48 BEAMS JUXTAPOSED

Conclusion

The Butler matrix is a versatile device. It can serve as a beam-forming network permitting volumetric beams to be generated that are orthogonal and independent, and each port will have the gain of the full array. Being passive and reciprocal, they can be used for both reception and transmission in an antenna array. The beams may be deployed simultaneously or sequentially depending on the application.

References

1. H.J. Riblet, "The Short-slot Hybrid Junction," *Proc.* IRE, February 1952.
2. J. Butler and R. Lowe, "Beam-forming Matrix Simplifies Design of Electrically Scanned Antennas," *Electronic Design*, April 1961.
3. W.A. Sanders, "The Butler Matrix Transponder," *COMSAT Technical Review,* Fall 1974.
4. J.P. Shelton and K.S. Kelleher, "Multiple Beams from Linear Arrays," *IRE Transactions on Antennas and Propagation*, March 1961.

5. T. MacNamara, "Simplified Design Procedure for Butler Matrices Incorporating 90° and 180° Hybrids," *IEE Proc. H, Microwave and Antenna and Propagation*, 1987, pp. 50–54.
6. J.R.F. Guy, "Proposal for the Use of Reflected Signals Through a Single Butler Matrix to Produce Multiple Beams From a Circular Array Antenna," *Electronic Letters*, 1985, pp. 209–211.
7. R. Levy, "A High Power X-band Butler Matrix," *Microwave Journal*, Vol. 11, No. 4, April 1968.
8. R.C. Hansen, *Microwave Scanning Antennas: Volume III*, Academic Press, New York, NY.
9. J.D. Kraus, *Antennas*, McGraw-Hill, New York, NY, 1950.

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